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by

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PREFACE

The torsional vibration absorber made its first introduction in England with the filing of the Lanchester patents. The device consisted essentially of a flywheel coupled to an engine shaft by friction discs. A torsional vibration in the shaft caused the flywheel to slip. This slipping resulted in an absorption of energy and a change in the natural frequency of the engine mass system. The amplitude of the vibration was consequently reduced.

A detailed theoretical analysis was first presented in 1926 by Major Carter in the Technical Reports of the British Aeronautical Research Committee. His classical analysis was premised on the laws of viscous friction. Although frequently applied, this theory was not strictly applicable to the numerous variations of the Lanchester damper in which energy was absorbed by dry frictional contact or by hydraulic resistance.

In 1930 Den Hartog and Ormondroyd made an experimental and theoretical investigation based on the assumption that only coulomb or dry friction is involved in the damping action. Although these theories give satisfactory results when applied to a particular type of damper, they are not entirely correct. The inherent difficulty in the development of a strictly exact theory lies in the fact that the damping force is a discontinuous function of the velocity.

One of the first purely viscous dampers in this country, which utilized a mass floating in a viscous fluid was designed in 1930 at the New York Naval Shipyard by Dashefsky and Captain Jensen. The viscous fluid used was a furniture glue. Lubricating oils were not satisfactory fluids because of the large variation of viscosity over comparatively

small temperature ranges. The development of the silicone oils in 1946 presented the designer with an excellent viscous medium which did not have the objectionable characteristics of the ordinary lubricating oils. In recent years O'Connor of the Houdaille Hershey Corporation (1947) and Georgian of the Nordberg Manufacturing Company (1949) published some of the very interesting results of their experiences in the design of dampers employing silicone fluids. The widespread adoption by the American engine manufacturing industry of viscous dampers with silicone fluids is evidence of their effectiveness.

The design of the tuned viscous torsional vibration damper employing silicone fluids has not been considered in the literature. Gatcombe and Ryder (1949) experimentally verified the existing theory utilizing a linear spring-mass system. The currently available theory of the analogous linear system is not strictly applicable to a torsional mass system where the absorber mass is coupled to the main mass system by an absorber spring and a shaft. In actual engine design practice this is the usual case.

There is, therefore, some justification for a theoretical and experimental investigation of the torsional mass system and the problem of tuning. It is the conviction of the writer that the tuned absorber has all the advantages of the simple viscous type plus the fact that it is more efficient.

The theoretical studies and experimental investigation described in this project were all performed at the United States Naval Postgraduate School, Annapolis, Maryland, during the period from October 1950 to April 1951.

The skillful machine work which contributed materially to the successful realization of this project was due to Mr. J. A. Cktavec of the mechanical force of the Postgraduate School. The viscosity-temperature curves of the various commercially available silicone oils were made available through the courtesy of the General Electric Company and the Dow Corning Corporation. A general acknowledgement is rightfully due to all those members of the faculty and the officers of the Postgraduate School who have directly or indirectly contributed to the author's educational background during the preceding three year course of instruction and who have made this project possible. A particular acknowledgement is due to Dr. Ernest K. Catcombe, whose unstinted helpfulness and guidance, greatly facilitated the accomplishment of the project.

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LIST OF ILLUSTRATIONS AND TABLES

TABLE OF SYMBOLS

a = Width of absorber spring, inches.

a_m = Width of absorber mass, inches.

A = Real part of the numerator of the generalized expression, $\frac{A + jB}{D + jF}$.

A_n = Real part of the numerator of the generalized expression, $\frac{A_n + jB_n}{D_n + jF_n}$, where the subscript, n, refers to the particular case number.

b = Length of the absorber spring, inches.

B = Imaginary part of the numerator of the generalized expression, $\frac{A + jB}{D + jF}$.

B_n = Imaginary part of the numerator of the generalized expression, $\frac{A_n + jB_n}{D_n + jF_n}$.

c = Damping constant of main mass system, lb-in.per radian per sec.

c_a = Damping constant of absorber system, lb-in. per radian per sec.

c_c = $2I_a\Omega$ = Pseudo critical damping constant of the absorber system, lb-in. per radian per sec.

c_i = Damping constant of the inside cylindrical surface of the absorber mass, lb-in. per radian per sec.

c_l = Damping constant of the lateral surfaces of the absorber mass, lb-in. per radian per sec.

c_o = Damping constant of the outside cylindrical surface of the absorber mass, lb-in. per radian per sec.

D = Real part of the denominator of the generalized expression, $\frac{A + jB}{D + jF}$.

D_n = Real part of the denominator of the generalized expression, $\frac{A_n + jB_n}{D_n + jF_n}$, where the subscript, n, refers to the particular case number.

e = Natural logarithm base.

E = Young's modulus of elasticity.

f = $\frac{\omega_a}{\Omega}$ = Tuning ratio, a dimensionless parameter.

f_1 = $\frac{\omega_1}{\Omega}$ = Tuning ratio, a dimensionless parameter.

f_2 = $\frac{\omega_2}{\Omega}$ = Tuning ratio, a dimensionless parameter.

g = $\frac{\omega}{\Omega}$ = Driving frequency ratio, a dimensionless parameter.

g_c = Common point driving frequency ratio, a dimensionless parameter.

h = $\frac{C_a}{C_c}$ = Damping ratio, a dimensionless parameter.

I = Moment of inertia of main mass system, lb-in-sec^2.

I_a = Moment of inertia of the absorber mass, lb-in-sec^2.

I_1 = Moment of inertia of the entire absorber assembly, lb-in-sec^2.

j = Complex operator, $\sqrt{-1}$.

K = Torsional spring constant of main mass shafting, in-lb per radian.

K_1 = Torsional spring constant of the absorber drive shaft, in-lb per radian.

K_2 = Equivalent torsional spring constant of the cantilever spring system, in-lb per radian.

$l = \frac{\omega_h}{\Omega}$ = Housing frequency ratio, a dimensionless parameter.

$m = \frac{I_h}{I}$ = Housing to main mass inertia ratio, a dimensionless parameter.

p = Force, pounds.

q = Thickness of absorber spring, inches.

q_o = Thickness of the oil film, inches.

R = Torque arm, inches.

R_i = Inside radius of absorber mass, inches.

R_o = Outside radius of absorber mass, inches.

S = Area, square inches.

V = Velocity, inches per second.

t = Time, seconds.

T_o = Maximum driving torque, in-lb.

T = Driving torque at any specified time, in-lb.

$u = \frac{I_a}{I}$ = Inertia ratio, a dimensionless parameter.

$u_1 = \frac{I_1}{I}$ = Inertia ratio, a dimensionless parameter.

δ = Deflection of absorber spring, inches.

θ = Angular displacement in radians of main mass from its initial rest position at a specified time, t.

θ_1 = Angular displacement in radians of the absorber housing from its initial rest position at a specified time, t.

θ_a = Angular displacement in radians of the absorber mass from its initial rest position at a specified time, t.

$\theta_{st} = \dfrac{T_o}{K}$ = Static angular displacement of the main mass in radians.

μ = Absolute viscosity, reyns or lbs per in^2 per sec.

ν = Kinematic viscosity, centistokes.

ϕ = Maximum amplitude of the angular displacement of the main mass in radians.

ϕ = Maximum amplitude of the angular displacement of the absorber housing in radians.

ϕ_a = Maximum amplitude of the angular displacement of the absorber mass in radians.

ϕ_c = Common point maximum amplitude of the angular displacement in radians.

ϕ_r = Maximum amplitude of the angular displacement of the absorber mass relative to the housing in radians.

$\Omega = \sqrt{\dfrac{K}{I}}$ = Undamped natural circular frequency of the main mass system, radians per sec.

ω = Forced circular frequency, radians per sec.

$\omega_a = \sqrt{\dfrac{K_2}{I}}$ = Undamped natural circular frequency of the absorber system, radians per sec.

$\omega_h = \sqrt{\dfrac{K_1}{I_h}}$ = Undamped natural circular frequency of the absorber housing, radians per sec.

$\omega_1 = \sqrt{\dfrac{K_1}{I_1}}$ = Undamped natural circular frequency of the absorber assembly, radians per sec.

$\omega_2 = \sqrt{\dfrac{K_1}{I_a}}$ = Undamped natural circular frequency of the absorber (Major Carter's general case), radians per sec.

ω_c = Common point circular frequency, radians per sec.

CHAPTER I

INTRODUCTION

1. Theoretical Investigation.

It is proposed in this project to correlate the theoretical investigations of Major Carter (2), Den Hartog (4), O'Connor (5), Georgian (6), and Gatcombe and Ryder (7) by the derivation of the general solution of the differential equations of motion of the elastic mass system represented in Fig. 6. The results obtained by these investigators shall be shown to be particular cases of this general solution.

All the necessary functions which define the various conditions of tuning and damping in terms of the parameters of the general solution shall be developed and applied to the test apparatus on the assumption that the effect of absorber shaft on tuning is negligible. By the application of Newton's law of viscous friction, the corresponding expression for the kinematic viscosity involving the fixed variables shall be derived.

It shall be demonstrated that, when the effect of the absorber shaft is considered in the analysis, the mathematical results are very unwieldy. For this reason the experimental investigation of the magnification function was undertaken.

2. Experimental Investigation.

In the experimental phase of this project it is proposed to design, build, and test a mechanical system that can be used in conjunction with the Westinghouse HI Vibration Equipment to investigate the possibility of verifying the theory of the tuned viscous vibration absorber and also the possible application of such a mechanical system to the solution of the theoretical problem. It is recognized at the outset that a pure torsional sinusoidal moment will be difficult to achieve experimentally with the apparatus at hand. It is hoped that the influence of the transverse vibration which will be introduced into the mechanical system of the HI vibration motor will be of negligible magnitude compared to the effect of the torsional vibration. In the event, that the transverse forces are of appreciable magnitude, a comparative study shall be made of the absorber operating with and without springs and using Silicones oil of varying viscosities with clearances of .005 and .010 inches.

CHAPTER II

1. Description of the Equivalent Elastic Systems.-

The elastic system of the testing apparatus for the tuned viscous vibration absorber is a system with three degrees of freedom and is represented diagramatically in Fig. 5. It consists of a torsional pendulum which is oscillated by an externally applied sinusoidal moment. The pendulum has a moment of inertia, I. Its shaft has a torsional spring constant, K, an angular displacement, θ, and a damping constant, c. This pendulum shall subsequently be referred to as the main mass.

The absorber system consists of a cylindrical shaft with a torsional spring constant, K_1, and an angular displacement, θ_1; the absorber housing (I_h) which is rigidly attached to the absorber driving shaft; and the absorber mass (I_a) coupled to the absorber driving shaft by a cantilever spring system (K_2) and viscous film with a damping constant, c_o.

The angular displacements of the absorber mass and the housing from an initial rest position are represented by the symbols, θ_a and θ_1. For the purpose of the mathematical analysis the cantilever spring system shall be considered equivalent to a section of shafting with a torsional spring constant, K_2, expressed in units of inch-pounds per radian. The elastic system then reduces to the system represented in Fig. 6.

For the condition of no damping when the absorber mass is locked in position by viscous forces, the problem becomes one of a two-mass elastic system. The total moment of inertia, I_t, represents the combined masses of the entire absorber assembly.

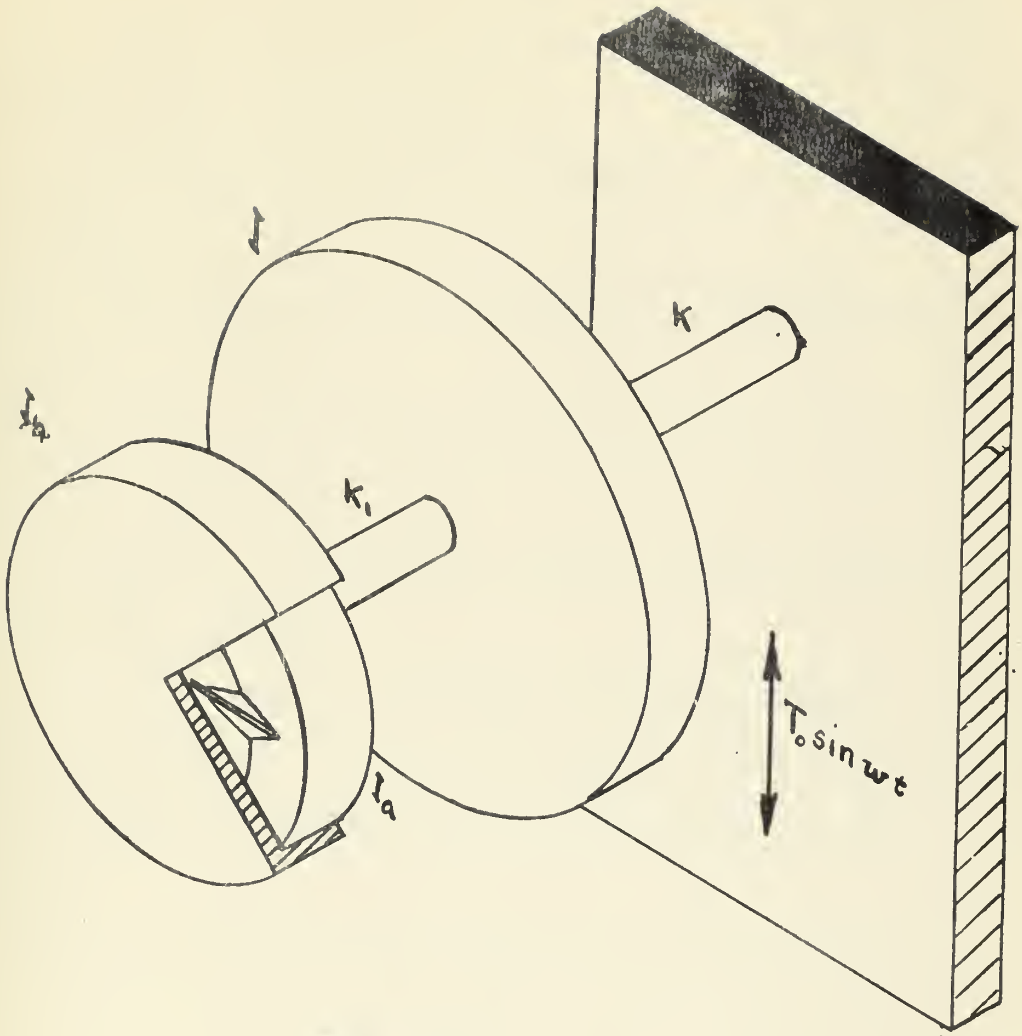


Figure 5.

A schematic diagram of the elastic system of the testing apparatus.

In the analysis of the elastic system of Fig. 6, the following assumptions shall be made:

a. The damping present is a linear function of the velocity.

b. The moments of inertia of the shafting, driving arm, and cantilever springs are negligible.

c. All frictional effects in the system, other than viscous, are negligible.

d. The amplitude of the exciting force is independent of its frequency.

e. The phase angle at the initial starting condition, $t = o$, is equal to zero.

f. Damping is present in the absorber assembly but not in absorber shaft bushings.

It should be noted that although the testing apparatus actually undergoes no rotation, the results of this somewhat more idealized testing are applicable to systems undergoing actual rotation.

Frequent references will be made in this analysis to the "rest position" of the various masses as a datum point in the derivation of the differential equations of motion and in the definition of the notation. Whenever a torsional vibration is superimposed upon the steady rotation of a system, as is done in the application of the theory of vibration absorbers, all displacements are measured from the "rest position" of the mass, neglecting the actual rotation. For the lack of a better phrase,

the "rest position" shall describe this angular location of a mass when it begins to vibrate torsionally.

2. Differential Equations of Motion for the General Case of the Tuned Viscous Vibration Absorber.-

At any time, t, the position of the system is completely defined by the three independent variables, θ, θ_1, and θ_a. These variables specify the angular displacements of the three masses from their initial rest position. The motions of the three masses, when they are displaced by an externally applied torque, are assumed to be harmonic and shall be expressed either as a sinusoidal function or complex function as the analysis dictates. The equations of motion of the main mass, the absorber housing, and the absorber mass respectively are:

$$\theta = \phi \sin \omega t \qquad \text{or} \quad \theta = \phi\, e^{j\omega t},$$

$$\theta_1 = \phi_1 \sin \omega t \qquad \text{or} \quad \theta_1 = \phi_1\, e^{j\omega t},$$

$$\theta_r = \phi_r \sin \omega t \qquad \text{or} \quad \theta_r = \phi_r\, e^{j\omega t},$$

where ϕ = the maximum amplitude of the angular displacement of the main mass measured from its initial rest position,

ϕ_1 = The maximum amplitude of the angular displacement of the absorber housing measured from its initial rest position,

ϕ_a = the maximum amplitude of the angular displacement of the absorber mass measured from its initial rest position.

When ϕ is a real quantity, these equations imply that the three masses, having been disturbed from their initial rest position

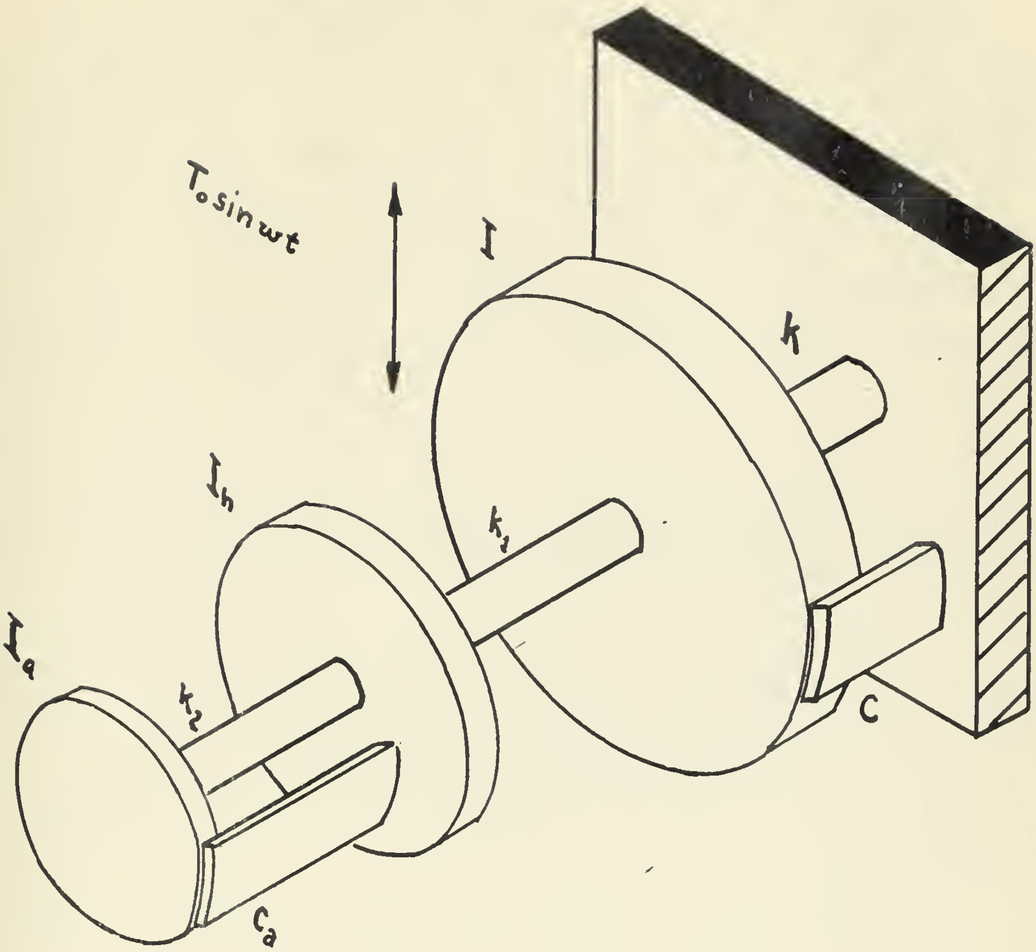


Figure 6.

A schematic diagram of the equivalent elastic system of the test apparatus.

a. Rotate sinusoidally with zero phase difference between each other,

b. Attain their maximum amplitudes at the same time, t, and

c. Pass through the points of their initial rest position at the same time, t.

However, when ϕ, ϕ_1 and ϕ_a are assumed to be complex, there is no inference that the phase difference between each mass is equal to zero. Such is the case when damping is present in the physical problem and is considered as an appreciable quantity in the mathematical analysis.

In the general case of Fig. 6, the moments acting on the main mass at the instant that it is rotated from its initial rest position are as follows:

$K\theta =$ Main torsional spring moment,

$K_1(\theta - \theta_1) =$ Absorber housing torsional spring moment,

$\dfrac{Cd\theta}{dt} =$ Main mass damping moment,

$I\,\dfrac{d^2\theta}{dt^2} =$ Moment due to the inertia of the main masses.

These moments oppose the applied sinusoidal torque, $T_0 \sin \omega t$. Applying Newton's second law, we obtain the differential equation of motion,

$$T_0 \sin \omega t = K\theta + K_1 (\theta - \theta_1) + c\,\frac{d\theta}{dt} + I\,\frac{d^2\theta}{dt^2}\ . \qquad (1)$$

The moments acting on the absorber housing mass are

$K_1(\theta_1 - \theta) =$ Torsional spring moment of the absorber driving shaft,

$K_2(\theta_1 - \theta_a) =$ Absorber mass torsional spring moment acting on the housing mass,

$C_a\left(\frac{d\theta_1}{dt} - \frac{d\theta_a}{dt}\right)$ = Damping moment of absorber acting on the housing mass, and

$I_h \frac{d^2\theta_1}{dt^2}$ = Moment due to the inertia of the absorber housing.

By the application of Newton's second law we obtain the following differential equation of motion,

$$K_1 (\theta_1 - \theta) + K_2(\theta_1 - \theta_a) + C_a \left(\frac{d\theta_1}{dt} - \frac{d\theta_a}{dt}\right) + I_h \frac{d^2\theta_1}{dt^2} = 0. \qquad (2)$$

The moments acting on the absorber mass are

$K_2 (\theta_a - \theta_1)$ = Absorber mass torsional spring moment,

$C_a \left(\frac{d\theta_a}{dt} - \frac{d\theta_1}{dt}\right)$ = Damping moment of the absorber, and

$I_a \frac{d^2\theta_a}{dt^2}$ = Moment due to the inertia of the absorber mass.

The differential equation of motion for the absorber mass becomes

$$K_2 (\theta_a - \theta_1) + C_a\left(\frac{d\theta_a}{dt} - \frac{d\theta_1}{dt}\right) + I_a \frac{d^2\theta_a}{dt^2} = 0 \quad . \qquad (3)$$

3. Steady State Solution of the Differential Equations for the General Case.-

For the steady state condition the impressed torque, T, and the angular displacements, θ, θ_1, and θ_a may be expressed in complex form as follows:

$$T = T_o\, e,^{j\omega t}$$

$$\theta = \phi\, e,^{j\omega t}$$

$$\theta_1 = \phi_2\, e,^{j\omega t}$$

$$\theta_a = \phi_a\, e,^{j\omega t}$$

where ϕ, ϕ_1, and ϕ_a are the maximum complex angular amplitudes.

When the above expressions and their first and second derivatives with respect to time are substituted in equations (1), (2), and (3), we obtain

$$T_o = K\phi + K_1\phi - K_1\phi_1 + j\omega c\ \phi - I\,\omega^2\phi, \tag{4}$$

$$K_1\phi_1 - K_1\phi + K_2\phi_1 - K_2\phi_a + j\omega C_a\phi_1 - j\omega C_a\phi_a - I_h\,\omega^2\phi_1 = 0, \tag{5}$$

$$K_2\phi_a - K_2\phi_1 + j\omega C_a\phi_a - j\omega C_a\phi_1 - I_a\,\omega^2\,\phi_a = 0. \tag{6}$$

Solving these equations for the three independent complex variables ϕ, ϕ_1, and ϕ_a, we arrive at the following expressions after considerable manipulation:

$$\phi = \frac{T_o(K_2 + j\omega C_a)^2 - T_o(K_1 + K_2 - I_h\omega^2 + j\omega C_a)(K_2 - I_a\omega^2 + j\omega C_a)}{K_1^2(K_2 - I_a\omega^2 + j\omega C_a) + (K_2 + j\omega C_a)^2(K + K_1 - I\omega^2 + j\omega C) - (K_1 + K_2 - I_h\omega^2 + j\omega C_a)(K + K_1 - I\omega^2 + j\omega C)(K_2 - I_a\omega^2 + j\omega C_a)} \quad (7)$$

$$\phi_1 = \frac{T_o(K_2 + j\omega C_a)^2(K + K_1 - I\omega^2 + j\omega C) - T_o(K_1 + K_2 - I_h\omega^2 + j\omega C_a)(K_2 - I_a\omega^2 + j\omega C_a)}{K_1^3(K_2 - I_a\omega^2 + j\omega C_a) + K_1(K_2 + j\omega C_a)^2(K + K_1 - I\omega^2 + j\omega C) - }$$

$$\frac{(K + K_1 - I\omega^2 + j\omega C)}{K_1(K_1 + K_2 - I_h\omega^2 + j\omega^2 + j\omega C_a)(K + K_1 - I\omega^2 + j\omega C)(K_2 - I_a\omega^2 + j\omega C_a)} - \frac{T_o}{K_1} . \quad (8)$$

$$\phi_a = \frac{T_o(K_2 + j\omega C_a)^3(K + K_1 - I\omega^2 + j\omega C) - T_o(K + K_1 - I\omega^2 + j\omega C)(K_2 + j\omega C_a)}{K_1^2(K_2 - I_a\omega^2 + j\omega C_a)^2 + K_1(K_2 - I_a\omega^2 + j\omega C_a)(K + K_1 - I\omega^2 + j\omega C)}$$

$$\frac{(K_1 + K_2 - I_h\omega^2 + j\omega C_a)(K_2 - I_a\omega^2 + j\omega C_a)}{(K_2 + j\omega C_a)^2 - K_1(K_1 + K_2 - I_h\omega^2 + j\omega C_a)(K + K_1 - I\omega^2 + j\omega C)(K_2 - I_a\omega^2 + j\omega C_a)^2}$$

$$- \frac{T_o\,(K_2 + j\omega C_a)}{K_1(K_2 - I_a\omega^2 + j\omega C_a)} . \quad (9)$$

After collecting all real terms and imaginary terms in the numerator and denominator, equation (7) assumes the form,

$$\frac{\phi}{T_o} = \frac{A + jB}{D + jF}, \qquad (10)$$

where $A = -K_1K_2 + \omega^2\left[I_a(K_1 + K_2) + I_hK_2\right] - I_hI_a\omega^4,$

$$B = \omega C_a\left[(I_h + I_a)\omega^2 - K_1\right],$$

$$D = -KK_1K_2 + \omega^2\left[I_hKK_2 + I_hK_1K_2 + I_aKK_1 + I_aKK_2 + I_aK_1K_2 + IK_1K_2 + K_1CC_a\right] - \omega^4\left[I_hI_aK_1 + I_hCC_a + I_aCC_a + I_hI_aK + II_hK_2 + II_aK_1 + II_aK_2\right] + \omega^6 II_hI_a,$$

$$F = -\omega(KK_1C_a + K_1K_2C) + \omega^3\left[I_hKC_a + I_hK_1C_a + I_hK_2C + I_aKC_a + I_aK_1C_a + I_aK_1C + I_aK_2C + IK_1C_a\right] - \omega^5\left[I_hI_aC + II_hC_a + II_aC_a\right].$$

By the application of the rules of complex numbers, it is readily seen that the absolute value of equation (10) becomes

$$\frac{\phi}{T_o} = \frac{A^2 + B^2}{D^2 + F^2} \qquad (11)$$

To transpose equation (11) into dimensionless form, we utilize the following dimensionless parameters and changes in notation:

$u = Ia/I$ = Absorber to main mass inertia ratio.

$f = \omega/\Omega$ = Tuning ratio.

$g = \omega/\Omega$ = Driving frequency ratio.

$h = Ca/Cc$ = Damping ratio

$m = Ih/I$ = Housing mass to main mass inertia ratio.

$\ell = \omega_h/\Omega$ = Housing frequency ratio.

$\Omega = \sqrt{K/I}$ = Undamped natural frequency of main mass.

$\omega_a = \sqrt{K_2/I_a}$ = Undamped natural frequency of absorber.

$\omega_h = \sqrt{K_1/I_n}$ = Undamped natural frequency of housing.

$\phi_{st} = \dfrac{T_o}{K}$ = Statical angular displacement of main mass system.

$C_c = 2I_a\Omega$ = Critical damping constant.

By dividing both the numerator and denominator of the right side of equation (11) by $(C_c \Omega\ K_1)^2$ and multiplying both sides of the equation by K^2, we convert it into the form,

$$\frac{\phi}{\phi_{st}} = \frac{A^2 + B^2}{D^2 + F^2}.$$

where

$$A = \frac{1}{2\ell^2}\left[\ell^2 (g^2 - f^2) + f^2 g^2 \left(1 + \frac{u}{m}\right) - g^4\right],$$

$$B = \frac{hg}{\ell^2}\left[g^2\left(1 + \frac{u}{m}\right) - \ell^2\right],$$

$$D = \frac{1}{2\ell^2}\left\{ ug^2 f^2 \ell^2 - \left[f^2 g^2 \left(1 + \frac{u}{m}\right) - g^4\right](g^2 - 1) \right.$$
$$\left. + \ell^2\left[1 - g^2(1+m)\right](g^2 - f^2) + 4hg^2 u \frac{C}{C_c}\left[\ell^2 - g^2\left(1 + \frac{u}{m}\right)\right]\right\},$$

$$F = \frac{hg}{\ell^2}\left\{\left[\ell^2 - g^2\left(1 + \frac{u}{m}\right)\right](g^2 - 1) + g^2\ell^2(u + m)\right.$$
$$\left. + u\frac{C}{C_a}\left[g^2 f^2\left(1 + \frac{u}{m}\right) + \ell^2(g^2 - f^2) - g^4\right]\right\}.$$

4. Particular Cases of the General Solution.-

In order to verify the general solution two of the particular cases, case 3 and 4, were derived directly from the applicable differential equations of motion. These independent derivations are briefly presented in sections (15)and (16) of Chapter II. In the following particular cases of the general solution, the angular displacement ratio or the magnification factor is expressed in the form,

$$\frac{\phi}{\phi_{st}} = \frac{A_h^2 + B_h^2}{D_h^2 + F_h^2} ,$$

Where the subscript, n, refers to the case number.

5. Case 1, No Damping in the Main Mass System. c = 0.-

In an experimental investigation of a linear mass system Ryder and Gatcombe (7), employing the identical vibration motor used in this project, demonstrated that the damping constant of the main mass system has a value equal to 1 1/2% of the critical value. This assumption of no damping is universally made in actual design applications of the theory. Substituting c = 0 in equation (11), we obtain

$$A_1 = \frac{1}{2\ell^2}\left[\ell^2(g^2-f^2) + f^2g^2(1+\frac{u}{m}) - g^4\right],$$

$$B_1 = \frac{hg}{\ell^2}\left[g^2\,(1+\frac{u}{m}) - \ell^2\right],$$

$$D_1 = \frac{1}{2\ell^2}\left\{u\ell^2f^2g^2 + \ell^2\left[1-g^2\,(1+m)\right](g^2-f^2) - \left[f^2g^2(1+\frac{u}{m}) - g^4\right](g^2-1)\right\}, \qquad (12)$$

$$F_1 = \frac{hg}{\ell^2}\left\{\left[\ell^2-g^2\,(1+\frac{u}{m})\right](g^2-1) + g^2\ell^2(u+m)\right\}.$$

6. Case 2, No Damping in Main System and Absorber Spring Inoperative: $c = 0$. $K_2 = 0$. $f = 0$.-

By equating K_2, the torsional spring constant of the absorber to zero, we obtain the solution for the case when the absorber springs fail in operation. The damping action now is due only to the viscous forces of the oil film. The general solution reduces to

$$A_2 = \frac{g^2}{2\ell^2}\left[\ell^2 - g^2\right],$$

$$B_2 = \frac{hg}{\ell^2}\left[g^2\left(1 + \frac{u}{m}\right) - \ell^2\right], \tag{13}$$

$$D_2 = \frac{1}{2\ell^2}\left\{g^4(g^2-1) + g^2\ell^2\left[1 - g^2(1+m)\right]\right\},$$

$$F_2 = \frac{hg}{\ell^2}\left\{\left[\ell^2 - g^2\left(1 + \frac{u}{m}\right)\right](g^2-1) + g^2\ell^2(u+m) - g^4\right\}.$$

7. Case 3, No Damping Present in the Main Mass and the Absorber Systems: $c = 0$. $c_a = 0$.-

The condition of no damping in the absorber system would occur if the oil accidentally escaped from the absorber housing. It is also assumed that the damping in the main mass system is negligible. As c and c_a approach zero, the general solution reduces to

$$A_3 = \frac{1}{2\ell^2}\left[\ell^2(g^2-f^2) + f^2g^2\left(1 + \frac{u}{m}\right) - g^4\right],$$

$$B_3 = 0 \tag{14}$$

$$D_3 = \frac{1}{2\ell^2}\left\{ug^2f^2\ell^2 - \left[f^2g^2\left(1 + \frac{u}{m}\right) - g^4\right](g^2-1) + \ell^2\left[1 - g^2(1+m)\right](g^2-f^2)\right.,$$

$$F_3 = 0.$$

8. Case 4. No Damping Present in the Main Mass System.

Infinite Damping in the Absorber: $c = 0.\ c_a = \infty$.-

When the absorber damping constant, c_a, approaches infinity, the absorber mass is virtually clamped to the housing assembly. This condition results when the damper is damaged, or if by some other means, the motion of the absorber mass relative to the housing is obstructed. To obtain the particular solution for this case, the numerator and denominator of the right side of equation (11) is divided by $(hg)^2$. As c approaches zero and c_a approaches infinity, the general solution reduces to:

$$A_4 = 0,$$

$$B_4 = \frac{1}{\ell^2}\left[g^2\left(1 + \frac{u}{m}\right) - \ell^2\right],$$

$$D_4 = 0, \tag{15}$$

$$F_4 = \frac{1}{\ell^2}\left\{\left[\ell^2 - g^2\left(1 + \frac{u}{m}\right)\right](g^2-1) + g^2\ell^2(u+m) - g^4\right\}.$$

9. Case 5. Damping Present in Main Mass System. Torsional Spring Constant of Absorber Shaft Equal to Infinity:

$K_1 = \infty\ .\ \ell = \infty$.-

If the absorber shafting is sufficiently short in length, the value of its torsional spring constant, K_1, may be assumed to approach infinity. The following particular solution results when the frequency ratio, l, is made to approach zero.

$$A_5 = \tfrac{1}{2}\left(g^2 - f^2\right), \tag{16}$$

$$B_5 = -(hg),$$
$$D_5 = \tfrac{1}{2}\left\{ug^2f^2 + (g^2-f^2)\left[1-g^2(1+m)\right] + 4hg^2u\frac{C}{C_c}\right\},$$
$$F_5 = hg\left\{(g^2-1) + g^2(u+m) + u\frac{C}{C_a}(g^2-f^2)\right\}.$$

The derivation of the above solution is presented by Gatcombe and Ryder in reference (7).

10. Case 6. No Damping in Main Mass System. Torsional Spring Constant of Absorber Shaft Equal to Infinity

$K_1 = \infty .\ \ell = \infty .\ c = 0.$-

If we further simplify case 5 by neglecting the damping of the main mass system, we arrive at the particular solution which has been developed by Den Hartog in reference (4),

$$A_6 = \tfrac{1}{2}(g^2-f^2),$$
$$B_6 = (-hg), \qquad (17)$$
$$D_6 = \tfrac{1}{2}\left\{ug^2f^2 + (g^2-f^2)\left[1-g^2(1+m)\right]\right\},$$
$$F_6 = hg\left[(g^2-1) + g^2(u+m)\right].$$

11. Case 7. Damping Present in the Main Mass System. No Absorber Springs. $K_2 = 0.\ f = 0.\ m = 0.$-

When the absorber torsional spring constant, K_2, and the absorber housing inertia ratio, m, approach zero, the solution reduces to Major Carter's general result (2), which is the theoretical basis for the design of the contemporary untuned viscous vibration absorber:

$$A_7 = \tfrac{1}{2}f_2^2g^2,$$
$$B_7 = hg\left[g^2-f_2^2\right],$$

$$D_7 = \frac{1}{2}\left[f_2^{\,2} g^2 \,(1-g^2) + 4hg^2 u \frac{C}{C_a} \,(f_2^{\,2}-g^2)\right], \tag{18}$$
$$F_7 = hg\left[(f_2^{\,2}-g^2)(g^2-1) + g^2 \ell^2 u + \frac{C}{C_a} u f_2^{\,2} g^2\right],$$

when

$$f_2 = \frac{\omega_2}{\Omega},$$
$$\omega_2 = \sqrt{\frac{K_1}{I_a}}.$$

12. Case 8. Damping Present in Main Mass System. Torsional Spring Constant of the Absorber Shaft is Equal to Infinity:

$K_1 = \infty$. $K_2 = 0$. $\ell = 0$. $f = 0$.

The general solution now reduces to Major Carter's special case which is developed in reference (2). The theory of the modern absorber, utilizing silicone oil as a viscous medium, is premised on the assumptions of this case. Reference (5) and (6) are cited as examples of the application of this theory.

$$A_8 = -1/2\, g^4,$$
$$B_8 = hg^3\left(1 + \frac{u}{m}\right), \tag{19}$$
$$D_8 = \frac{1}{2}\left[g^4(g^2-1) - 4\, hu \frac{C}{C_c}\, g^4\left(1+\frac{u}{m}\right)\right],$$
$$F_8 = -hg^3(g^2-1)\left(1+\frac{u}{m}\right) - u\frac{C}{C_a}\, g^4$$

13. Case 9. Simple Torsional Pendulum. $c = 0$. $c_a = \infty$. $K_1 = \infty$.

The general solution reduces to the case of a simple torsional pendulum when c_a and K_1 are made to approach infinity and c is equated to zero. The total moment of inertia of the pendulum will

then be the sum of the three moments of inertia, I, I_a, and I_h.

$$\frac{\phi}{\phi_{st}} = \frac{-1}{\left[g^2(u+m+1) - 1 \right]} \tag{20}$$

14. Discussion of Magnification Factor Function.-

If we consider g and h as independent variables and hold u, m, l, and f constant in equation (11), then the magnification factor, $\frac{\phi}{\phi_{st}}$, can be represented as a surface. The partial derivative of $\frac{\phi}{\phi_{st}}$ with respect to g, when it is equated to zero, will give the locus of the maximum values of the magnification. By taking the partial derivative with respect to h of $\frac{\phi}{\phi_{st}}$ and setting it equal to zero, we obtain the locus of the minimum values of the magnification. To find the magnification which has the minimum value on the locus of the maxima, the expression for g obtained from $\frac{\partial \frac{\phi}{\phi_{st}}}{\partial h} = 0$ is substituted in the function, $\frac{\phi}{\phi_{st}}$, equation (11). In order to arrive at a value of h, which will result in optimum tuning for a given value of u, m. and l, we substitute this same expression for g in the equation evolved from $\frac{\partial \frac{\phi}{\phi_{st}}}{\partial g} = 0$.

The partial derivative of $\frac{\phi}{\phi_{st}}$ with respect to h, when it is equated to zero, will yield the following biquadratic equation in g^2:

$$g^8 - g^6\left[\ell^2\left(\frac{m}{m+u} + m + 1\right) + f^2\frac{(m+u)}{m} + 1\right]$$

$$+ g^4\left\{\ell^4\frac{m(1+m+\frac{1}{2}u)}{m+u} + \ell^2 f^2\left[u\left(\frac{m}{m+u} + 1\right) + 2 + \frac{m^2}{m+u}\right] + \right.$$

$$\ell^2\left(\frac{m}{m+u}+1\right)+f^2\left(\frac{m+u}{m}\right)\Big\} \tag{21}$$

$$-g^2\Big\{\ell^4\left[\frac{m}{m+u}+f^2\left(\frac{m}{m+u}+m\right)\right]+2\ell^2f^2\Big\}$$

$$+\,2\ell^4f^2\left(\frac{m}{m+u}\right)=0$$

The solution of the above equation for g will determine the value of g for the maximum values of $\frac{\phi}{\phi_{st}}$, when $\frac{\phi}{\phi_{st}}$ is plotted against g for any fixed value of u, m, l, and f. When we set the partial derivative of $\frac{\phi}{\phi_{st}}$ with respect to g equal to zero, an equation of the eighth degree in g^2 results. The expression for g thus obtained is too unwieldy to be of any practical utility. It is for this reason, therefore, that an experimental investigation of this function was deemed necessary.

15. Differential Equations of Motion for the Case of No Damping in the Absorber and Main Mass System (Absorber locked).-

Let us consider the elastic system shown in Fig. 6, when the viscous forces present are sufficient to prevent any oscillation of the absorber. The moments acting on the main mass are the torsional spring moments of the main mass shaft and the absorber driving shaft. The moment acting on the absorber assembly mass is simply the torsional spring moment of the absorber driving shaft. As a further simplification of the elastic system, it is assumed that the damping within the main mass system can be neglected. By application of Newton's second law, the differential equations of motion are:

$$I_1\frac{d^2\theta_1}{dt^2}+K_1(\theta_1-\theta)=0, \tag{22}$$

$$I \frac{d^2\theta}{dt^2} + K\theta + K_1(\theta - \theta_1) = T_o \sin\omega t. \tag{23}$$

Let us assume that the frequency of the angular displacements is equal to and in phase with the impressed frequency, ω, the condition of resonance. The angular displacements, therefore, can be expressed as harmonic functions of the impressed frequency.

$$\theta = \phi \sin\omega t,$$

$$\theta_1 = \phi_1 \sin\omega t.$$

Taking the second derivative with respect to time of θ and θ_1, we obtain:

$$\frac{d^2\theta}{dt^2} = -\phi\omega^2 \sin\omega t,$$

$$\frac{d^2\theta_1}{dt^2} = -\phi_1\omega^2 \sin\omega t.$$

By substitution of the above expression for θ, θ_1, $\frac{d^2\theta}{dt^2}$, and $\frac{d^2\theta_1}{dt^2}$ in the differential equations of motion, the following equations in terms of the maximum amplitudes result after cancellation of the common factor, sin wt:

$$I\omega^2\phi = K\phi + K_1(\phi - \phi_1) - T_o, \tag{24}$$

$$I_1\omega^2\phi_1 = K_1 (\phi_1 - \phi). \tag{25}$$

Solving these simultaneous equations for ϕ and ϕ_1, there results

$$\phi = \left(\frac{K_1 - I_1\omega^2}{(K_1 - I_1\omega^2)\left[K - I\omega^2 + K_1\right] - K_1^2} \right) T_o \ , \tag{26}$$

$$\phi_1 = \left(\frac{K_1}{(K_1 - I_1 \omega^2)\left[K - I\omega^2 + K_1\right] - K_1^2} \right) T_o \qquad (27)$$

The above expressions for ϕ and ϕ_1 have their maximum value when the common denominator is equated to zero.

$$(K_1 - I_1 \omega^2)\left[K - I\omega^2 + K_1\right] - K_1^2 = 0 \qquad (28)$$

After expanding and collecting terms, we obtain a quadratic equation,

$$\frac{I}{K}\,\frac{I_1}{K_1}\,\omega^4 - \left(\frac{I}{K} + \frac{I_1}{K_1} + \frac{I_1}{K}\right)\omega^2 + 1 = 0. \qquad (29)$$

The solution of the above quadratic equation results in the following expression for ω^2,

$$\omega^2 = \tfrac{1}{2}\left(\frac{K_1}{I_1} + \frac{K}{I} + \frac{K_1}{I}\right) \pm \sqrt{\tfrac{1}{4}\left[\left(\frac{K_1}{I_1}\right)^2 + \left(\frac{K}{I}\right)^2 + \left(\frac{K_1}{I}\right)^2 + \tfrac{1}{2}\left(\frac{KK_1}{II_1} + \frac{K_1^2}{II_1} + \frac{KK_1}{I^2}\right)\right] - \frac{KK_1}{II_1}} \qquad (30)$$

The above equation gives us the expression for the natural frequency. It is to be observed that the minimum amplitude of angular displacement of the main mass system will occur when the natural frequency of the absorber assembly is equal to the impressed frequency or

$$\omega = \sqrt{\frac{K_1}{I_1}}\ .$$

To transpose equation (30) into dimensionless form, we utilize the following dimensionless parameters and changes in notation:

$\mu_1 = \dfrac{I_n + I_a}{I} =$ Absorber assembly to main mass inertia ratio.

$f_1 = \omega_1/\Omega_n =$ Tuning ratio.

$\omega_1 = \sqrt{\dfrac{K_1}{I_n + I_a}} =$ Undamped circular frequency of the absorber assembly.

$\Omega_n = \sqrt{\frac{K}{I}}$ = Undamped circular frequency of main mass system.

$\phi_{st} = T_o/K$ = Statical angular displacement of main mass system.

$g = \omega/\Omega_n$ = Driving frequency ratio.

By dividing both the numerator and denominator of the right side of equation (30) by $\frac{\Omega_n^2 K_1}{\omega_1^2}$ and multiplying both sides of the equation by K, we change it into the dimensionless form,

$$\frac{\phi}{\phi_{st}} = \frac{g^2 - f_1^2}{u_1 g^2 f_1^2 - (g^2 - f_1^2)(g^2 - 1)} . \tag{31}$$

The above equation, when transformed into the notation of the general solution by letting $\omega_1 = \sqrt{\frac{K_1}{I_h + I_a}}$ and $u_1 = \frac{I_h + I_a}{I}$, becomes

$$\frac{\phi}{\phi_{st}} = \frac{g^2(1 + \frac{u}{m}) - \ell^2}{\ell^2 g^2 (u + m) - \left[g^2(1 + \frac{u}{m}) - \ell^2\right](g^2 - 1)} . \tag{32}$$

This result corresponds to case 4 of the general solution.

16. Differential Equations of Motion for the Case of No Damping in the Absorber and Main Mass System (absorber Mass in Motion Relative to Housing).-

If we assume that there is no damping in the absorber and the main mass system and that the absorber mass is free to oscillate, our system reduces to a simple one involving three masses. This condition would occur in the physical system if the oil accidentally escaped from the absorber housing, and if we made the further simplifying assumption that the damping in the main mass system is

negligible. The differential equations of motion are:

$$K\theta + K_1(\theta - \theta_1) + I \frac{d^2\theta}{dt^2} = T_o \sin\omega t , \tag{33}$$

$$K_1(\theta_1-\theta)+K_2(\theta_1-\theta_a)+ I_h \frac{d^2\theta_1}{dt^2} = 0, \tag{34}$$

$$K_2(\theta_a - \theta_1)+ I_a \frac{d^2\theta_a}{dt^2} = 0 . \tag{35}$$

Solving the above equations for the three independent variables, θ, θ_1, and θ_2, and equating the common denominator of each expression to zero, we arrive at a cubic equation in ω^2,

$$\omega^6 II_aI_h - \omega^4\left[II_aK_1+ I_aI_hK_1 + II_aK_2 + I_hI_aK\right] \tag{36}$$
$$+\omega^2\left[IK_1K_2 + I_h(KK_2+ K_1K_2) + I_a(KK_1 + KK_2 + K_1K_2)\right]$$
$$-KK_1K_2 = 0.$$

The solution of the above frequency equation gives us an approximate method for determining the normal modes of oscillation of the general system.

The expression for θ obtained from the simultaneous solution differential equations of motion is:

$$\frac{\phi}{T_o} = \frac{(k_1- \omega^2 I_h + K_2)(K_2 - \omega^2 I_a) - K_2^{\,2}}{(K + K_1 - I\omega^2)\left[(K_1- \omega^2 I_h + K_2)(K_2- \omega^2 I_a) - K_2^{\,2}\right] - K_1^{\,2}(K_2- \omega^2 I_a)} . \tag{37}$$

By dividing both the numerator and denominator of the right side of the above expression by K_2K_1 and multiplying both sides by K, we transform it into the dimensionless form,

$$\frac{\phi}{\phi_{st}} = \frac{\ell^2(g^2-f^2) + f^2g^2(1+\frac{u}{m}) - g^4}{u\ell^2f^2g^2 + \ell^2\left[1-g^2(1+m)\right](g^2-f^2) - \left[f^2g^2(1+\frac{u}{m}) - g^4\right](g^2-1)} . \tag{38}$$

This is the identical result that is obtained from the special case of the general solution when the dimensionless parameters C/c_c and h are equated to zero.

17. Theoretical Consideration in the Design of the Test Apparatus.-

It was desired that the test apparatus would permit the investigation of the operation of an experimental absorber under the following conditions of damping:

(A) Condition 1. Optimum Damping without Tuning (No Absorber Springs).

The equations which define this condition of damping are derived from case 8 of the general solution. For methods by which they can be independently derived, the reader is referred to references (2), (4), (5), and (6). These equations are summarily listed below using the notation as defined in this text:

$$f = 0, \tag{39}$$

$$g_c^2 = \frac{2}{2 + 2m + u}, \tag{40}$$

$$\left(\frac{C_a}{C_c}\right)^2 = \frac{m+1}{2(2+2m+u)(1+m+u)}, \tag{41}$$

$$\left(\frac{\phi_{r,c}}{\phi_{st}}\right)^2 = \frac{\phi_c}{\phi_{st}} \quad \frac{(m+1)}{2\,u\,g_c \frac{C_a}{C_c}}. \tag{42}$$

The subscript, c, is introduced into the notation to earmark the specific "common point" value of a variable. The common points are the points which the curves of the magnification ratio versus impressed frequency ratio will have in common plotted for various values of the damping ratio. These values are of particular

interest to the designer because they represent a maximum value at the optimum condition of damping.

(B) Condition 2. Damping with Constant Tuning, Effect of the Torsional Spring Constant of the Absorber Shaft is Negligible.

The equations which define this condition of damping are derived from Case 6 of the general solution. For the manner in which they are developed, the reader is referred to Den Hartog's analysis in reference (4). Using the notation on this text, they are:

$$f^2 = \frac{1}{1+m}, \tag{43}$$

$$\left(\frac{C_a}{C_c}\right)^2 = \frac{u(3+u+3m)\left(1+\sqrt{\frac{u}{2+u+2m}}\right)}{8(1+m)^2(1+m+u)}, \tag{44}$$

$$\frac{\phi_c}{\phi_s} = \frac{(1+m)}{-u+(1+m+u)\sqrt{\frac{u}{2+u+2m}}}, \tag{45}$$

$$\left(\frac{\phi_{r.c}}{\phi_{st}}\right)^2 = \frac{\phi_c}{\phi_{st}} \quad \frac{(m+1)}{2\,u\,g_c\,\frac{C_a}{C_c}}. \tag{46}$$

(C) Condition 3. Optimum Tuning. Effect Constant of the Absorber Shaft is Negligible.

Theequations which define this condition of damping are derived from Case 6 of the general solution. For the detailed manner in which they are developed, the reader is again referred to Den Hartog's analysis in reference (4).

$$f = \frac{\sqrt{1+m}}{1+u+m}, \tag{47}$$

$$\left(\frac{C_a}{C_c}\right)^2 = \frac{3u(1+m)}{8(1+u+m)^3}, \tag{48}$$

$$\frac{\phi_c}{\phi_{st}} = \sqrt{1+\frac{2}{u}(1+m)}, \tag{49}$$

$$\frac{\phi_{r\,c}}{\phi_{st}} = \left(\frac{\phi_c}{\phi_{st}}\right)\frac{(m+1)}{2ug\frac{C_a}{C_c}}. \tag{50}$$

To make the investigation general it was required that these conditions of damping be obtained over a wide range of values of the inertia ratio, u. For reasons of simplicity of construction it was decided

(a) to make the absorber mass inertia, I_a, and the absorber torsional spring constant fixed quantities and

(b) to attain the desired condition of damping by varying the value of the main mass inertia, I, the main torsional spring constant, K, and the absorber shaft torsional spring constant, K_1.

18. Main Shaft System.-

At any desired condition of tuning the main mass torsional spring constant may be expressed as a function of the absorber spring constant, K_2, and the inertia ratio, ü. The following functions are obtained from equations (43)and (47):

Condition 2. Constant Tuning.

$$K = \frac{K_a}{u}(1+m). \tag{51}$$

Condition 3. Optimum Tuning.

$$K = \frac{K_a(1+u+m)^2}{u(m+1)} . \tag{52}$$

To obtain Condition 1 of damping, the problem of tuning is not involved. However, to obtain optimum damping the main spring constant is varied so that the following function of the inertia ratio and the common point circular frequency is satisfied:

$$\omega_c = \sqrt{\frac{K}{I} \frac{2}{(2+u+2m)}} . \tag{53}$$

19. The Absorber Damping Constant.-

For a particular value of the inertia ratio, the value of the main spring constant is fixed for a specified condition of damping. The value in the test apparatus is obtained by positioning the fixed end support along the shaft length. The absorber damping constant must now be adjusted to satisfy the condition equations. Since this is accomplished in the test apparatus by the selection of an oil of suitable viscosity, the next step in the experimental procedure is to evaluate the viscosity in terms of the fixed quantities for the specified conditions of damping.

If both sides of the equation expressing Newton's law of viscous friction are multiplied by the torque arm, R, we obtain

$$\frac{P}{S} R = \frac{V\mu}{q_o} R, \tag{54}$$

where P = Force, lbs,

R = Torque arm, inches,

μ = Absolute viscosity, reyns,

$V =$ Velocity, in/sec,

$q_o =$ Thickness of the oil film, inches,

$S =$ Area, in^2.

By definition, the absorber damping constant is equal to the torque divided by the circular frequency.

$$C_a = \frac{PR}{a} . \tag{55}$$

By combining equations (54) and (55), the absorber constant can be expressed as a function of the absolute viscosity.

$$C_a = \frac{\mu R^2 S}{q_o} . \tag{56}$$

Referring to Fig. 7, the damping coefficient of the absorber may be taken as the sum of the inside surface damping coefficient, c_i, the outside surface damping coefficient, c_o, and the damping of the lateral surfaces, c_l. For the sake of simplicity it is assumed that the absorber mass is a hollow circular cylinder.

Using the general expression (56), we obtain the following equations for the component coefficients:

$$C_i = \frac{2\pi\mu R_i^3 a_m}{q_o} , \tag{57}$$

$$C_o = \frac{2\pi\mu R_o^3 a_m}{q_o} , \tag{58}$$

$$C = \frac{\pi\mu}{q_o} (R_o^4 - R_i^4) . \tag{59}$$

Summing up the individual coefficients, the total absorber damping coefficient becomes

$$C_a = \frac{\pi\mu}{q_o} \left[(R_o^4 - R_i^4) + 2 a_m (R_o^3 + R_i^3) \right] . \tag{60}$$

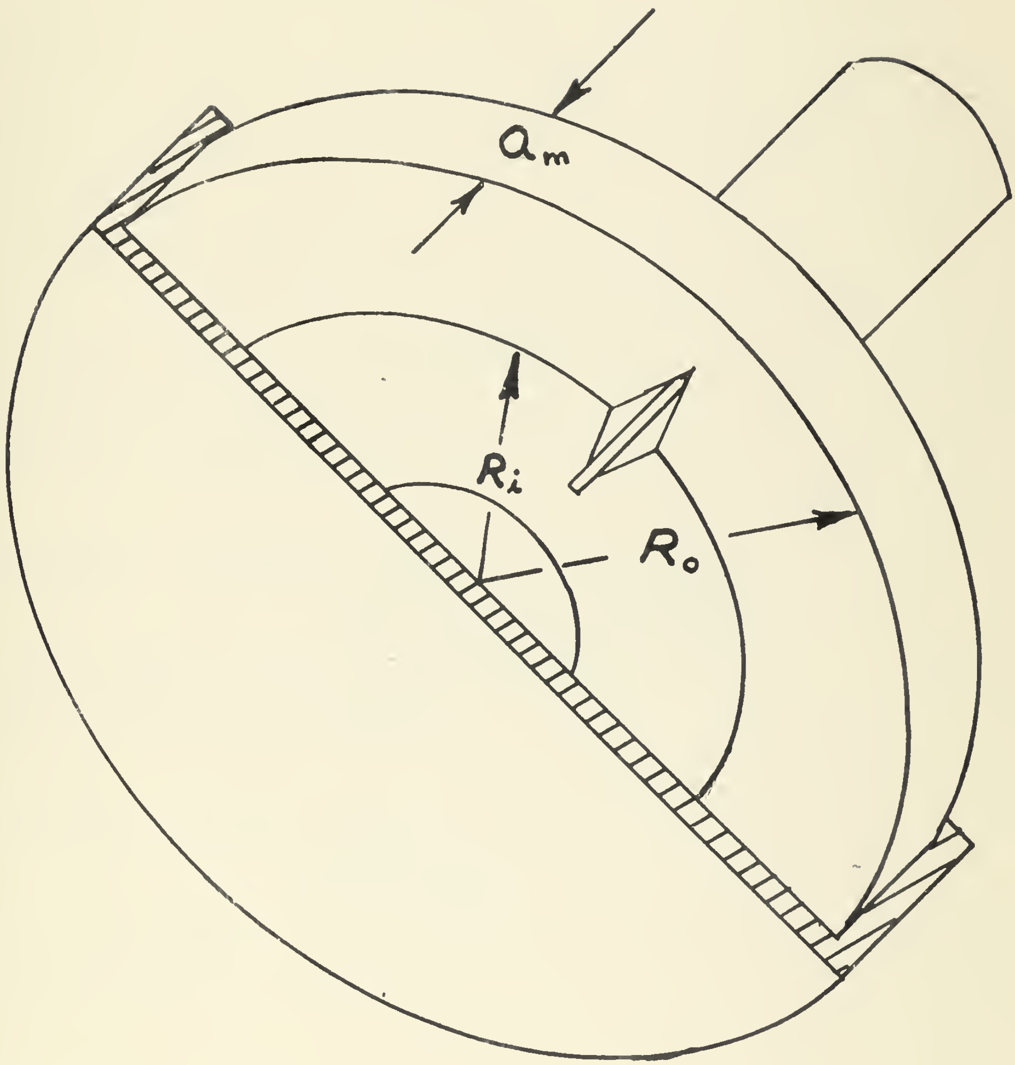


Figure 7

Half section view of the tuned torsional vibration absorber.

If we equate the damping condition equations (41), (44), and (48) to equation (60) and solve for the absolute viscosity, we can thereby evaluate the viscosity of the oil required for the specific conditions of damping. This expression will give us the viscosity in absolute units, reyns. To change this expression to the conventional units of centistokes, the expression must be multiplied by the conversion factor, 6.9×10^6 and divided by the specific gravity of the oil, .97. In units of centistokes, the expressions for the kinematic viscosity, ν, are as follows:

Condition 1. Optimum Damping without Tuning.

$$\nu = \frac{3.202 \times 10^6 \, q_o I(1+m)\sqrt{\dfrac{K\ (m+1)}{I(2+2m+u)(1+m+u)}}}{\left[(R_o^4 - R_i^4) + 2a_m(R_o^3 + R_i^3)\right]} \tag{61}$$

Condition 2. Constant Tuning.

$$\nu = \frac{1.6013 \times 10^6 q_o \ I(1+m)\sqrt{\dfrac{Ku(3+u+3m)\left(1+\sqrt{\dfrac{u}{2+u+2m}}\right)}{I(1+m)^2(1+m+u)}}}{\left[(R_o^4 - R_i^4) + 2a_m\ (R_o^3 + R_i^3)\right]} \tag{62}$$

Comdition 3. Optimum Tuning.

$$\nu = \frac{2.773 \times 10^6 q_o \ I(1+m)\sqrt{\dfrac{Ku\ (1+m)}{I(1+u+m)^3}}}{\left[(R_o^4 - R_i^4) + 2a_m\ (R_o^3 + R_i^3)\right]} \tag{63}$$

20. Absorber Spring Design .-

For the purpose of designing the absorber springs, the conventional deflection formula for a cantilever beam fixed at one end was used.

$$\delta = \frac{4Pb^3}{Eaq^3} , \tag{64}$$

where a = Width of absorber spring inches,

b = Length of absorber spring inches,

q = Thickness of absorber spring, inches,

P = Force, pounds,

E = Young's modulus of elasticity,

δ = Deflection, inches.

If it is assumed that a unit force acts on the end of the spring or $P = 1$ pound, the deflection is now expressed in units of inches per pound. To convert the equation to one in units of torque per radian, the reciprocal of the deflection, δ, is multiplied by the factor, R^2.

$$K_2 = \frac{Eaq^3R^2}{4b^3}, \tag{65}$$

where K_2 = Torsional spring constant of absorber, inch-pounds per radian,

R = Radius of torque arm, inches.

21. Design of the Absorber Mass.-

The dimensions of the absorber mass were arbitrarily selected to give a value of the moment of inertia which would not necessitate making the moment of inertia of the main mass unduly large when the inertia ratio, u, equals 1/20. The inside radius of the absorber, R_i, was fixed by the length of the absorber was determined from the equation,

$$I_a = \frac{\pi q_a \rho}{2(386)} (R_o^4 - R_i^4), \tag{66}$$

where I_a = Moment of inertia of absorber, lb-in-sec^2,

q_a = Thickness of absorber, inches,

ρ = Density, lbs/in^3,

386 = Acceleration of gravity, in/sec^2,

R_o = Outside radius of absorber, inches,

R_i = Inside radius of absorber, inches.

CHAPTER 3

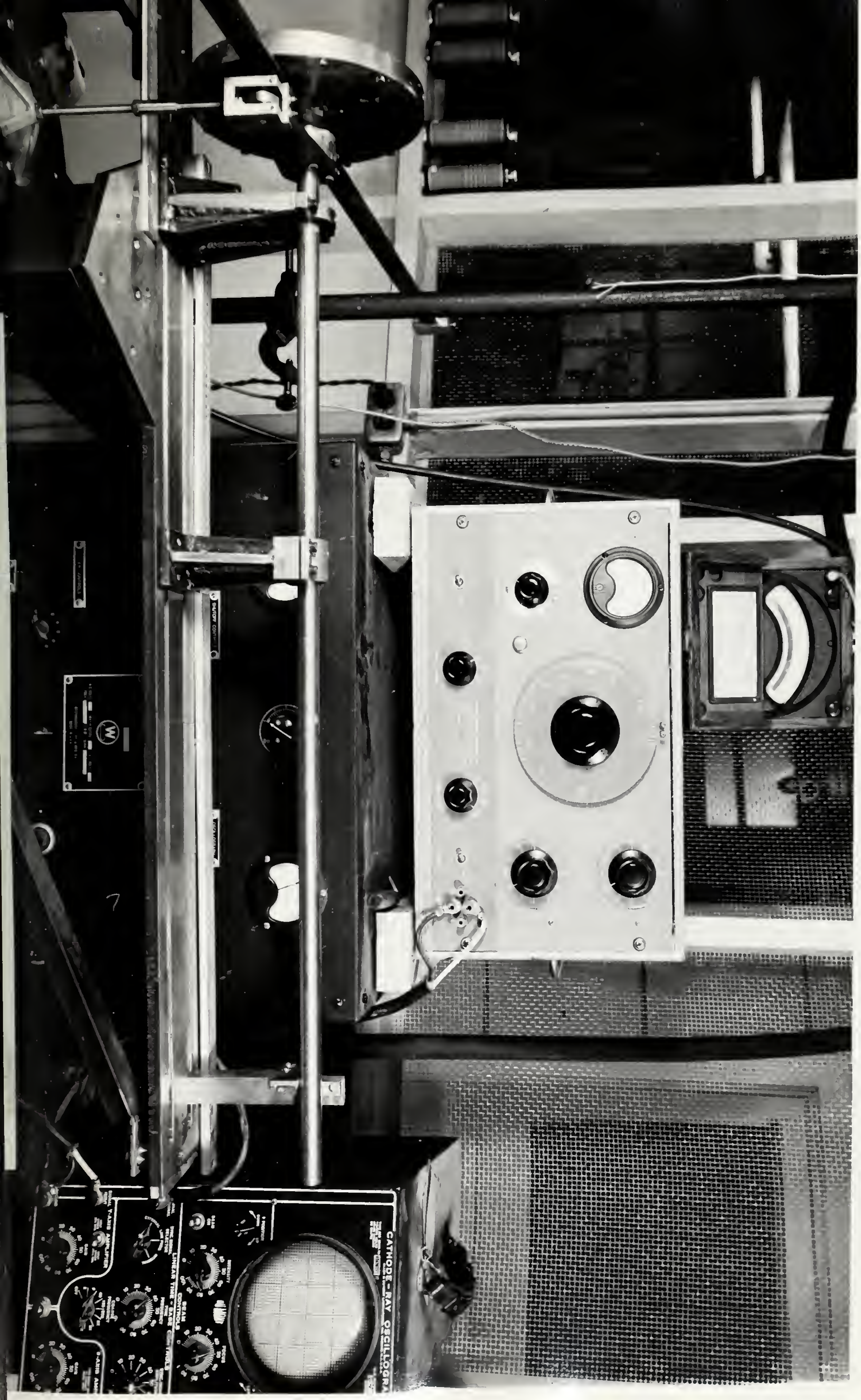
1. Description of Testing Apparatus.

The testing apparatus illustrated in figure (9) consists of a torsional pendulum which is oscillated by a Westinghouse vibration motor. The absorber assembly is attached to the free end of the testing shaft by two #3 standard tapered pins. The shaft is 40 inches in length to permit a wide range of variation in the torsional spring constant of the pendulum. It is varied by positioning the fixed end support along the guide plate to obtain a desired value of the spring constant. The shaft is also sufficiently long to permit the investigation of the effect of the stiffness of the coupling shaft on tuning.

The torsional pendulum or main mass system is connected to the vibration motor by a 5 inch driving arm. The linear motion of the motor is converted by the driving arm to rotational motion. The spring suspension of the vibration motor permits a maximum amplitude of 1/16 of an inch from the rest position of the coil. At maximum amplitude the lateral displacement of the driving coil will be approximately .0005 of an inch.

The moment of inertia of the main mass system can be varied by the attachment of weights to the driving arm locking piece. Any desired value of u, the ratio of absorber inertia to the inertia of the main mass system, can therefore be obtained by attaching these weights.

The testing apparatus permits us to vary the value of u, the inertia ratio, and K, the torsional spring constant of the main



system. The value of the absorber spring constant is fixed by the original design of the system. To tune the absorber for a given value of u, the torsional spring constant of the main mass system, K, can be varied by positioning the fixed end support along the guide plate.

2. Vibration Fatigue Equipment.

The Westinghouse Fatigue Equipment, Type HI-40 Watt Amplifier-Exciter, is designed to oscillate linearly mechanical systems over a frequency range of 20 to 1600 cycles per second. A 120 volt, direct current power supply was used to energize the coil. Power for the other units of the equipment was obtained from a 115 volt, 60 cycle alternating current supply.

A Hewlett-Packard audio signal generator, model 205-A was used with the apparatus. It consists primarily of a resistance-tuned oscillator combined with an output meter, attenuator and an impedance matching system. The output of this oscillator is amplified by a 40 watt Amplifier-Exciter unit and is supplied to the vibration motor.

3. The Vibration Motor

The vibration motor is a component of the Westinghouse vibration fatigue equipment. It consists of a coil which is free to move vertically in a uniform magnetic field. By means of a spider, this coil is attached to a drive rod. The coil vibrates in this magnetic field when it is energized by the amplified output of a audio frequency oscillator. The spring suspension allows a maximum angular displacement of 1/80 of a radian measured from the rest position of the coil.

For the purpose of vibrating mechanical systems at their natural frequencies, a pick-up coil wound on the top of the drive coil can feed back to the input of the 40 watt amplifier to maintain a self-excited oscillation. This pick-up coil was used to supply a signal to the oscillograph for observing and recording the motion of the elastic system.

4. Measurement of Displacements.

The angular displacements of the absorber mass and the driving arm were determined by measuring the chord lengths with a Gaertner traveling microscopes mounted on the guide plate. A General Radio Company, strobolux, type 648-A, triggered by a strobotac, type 631-13, was used to slow the mass down. The strobolux can be adjusted to any value from 10 to 100 cycles per second with an accuracy of $\pm$ 1% on the dial reading.

The amplitudes as measured are from the peak to peak position and were obtained by focusing the telescope on scribe marks located on a 5 inch radius arm. Readings were taken at each peak position. The difference between these two readings divided by two is the amplitude.

5. The Vibration Absorber.

The details of the vibration absorber are furnished in plans enclosed in an envelope attached to the back cover. The absorber consists of a cadmium plated steel cylinder 1/2 inch thick with an outside diameter of 7 1/2 inches and an inside diameter of 3 1/2 inches. The absorber is completely enclosed by the absorber housing which consists of two coverplates, a cylindrical spacer, and a center piece. The designed clearance between the housing and the absorber

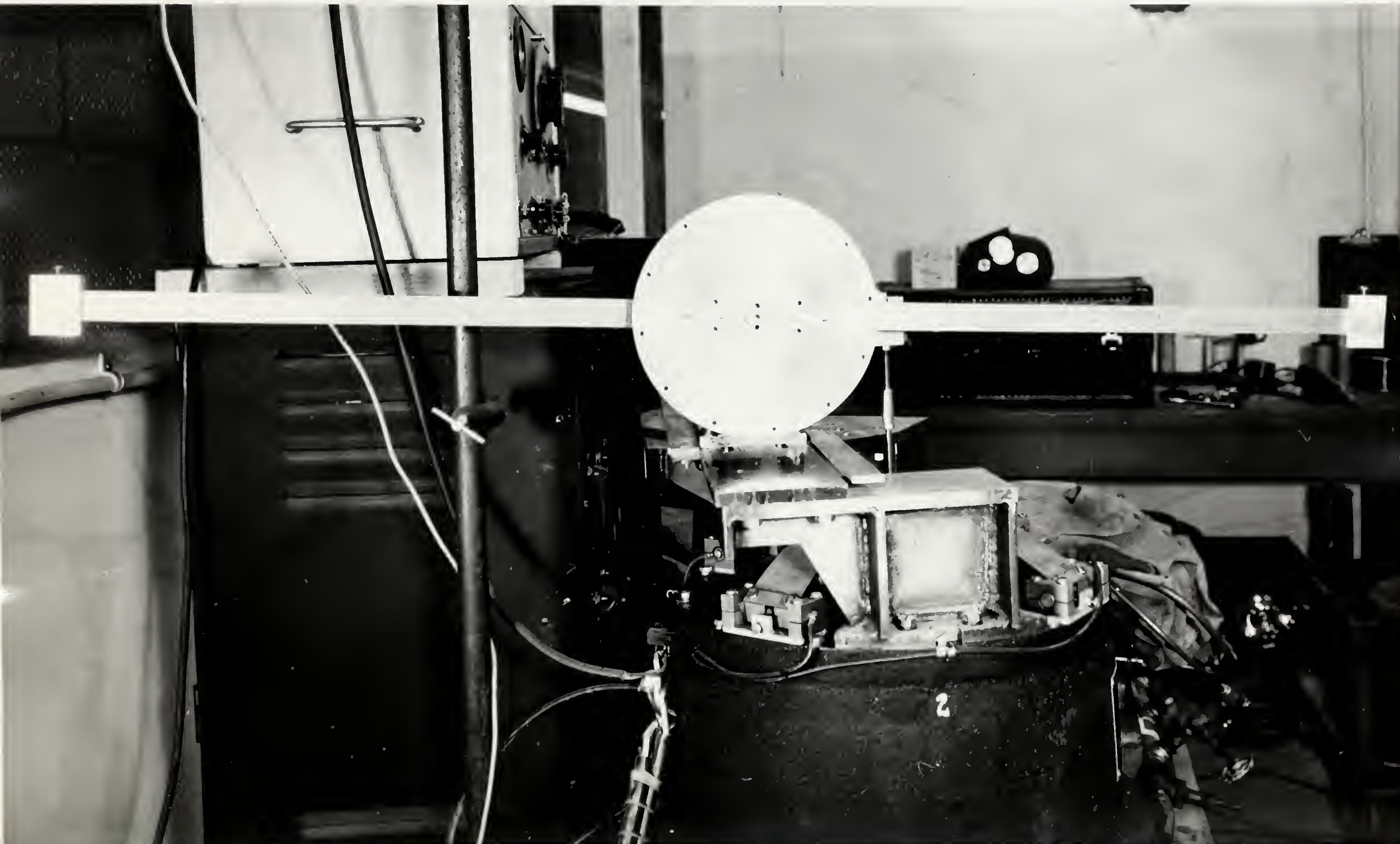


Fig. 10

is .005 inches. This permits the absorber to float in the silicone oil film. The absorber is essentially a flywheel which is coupled to a shaft by a viscous film and two cantilever springs.

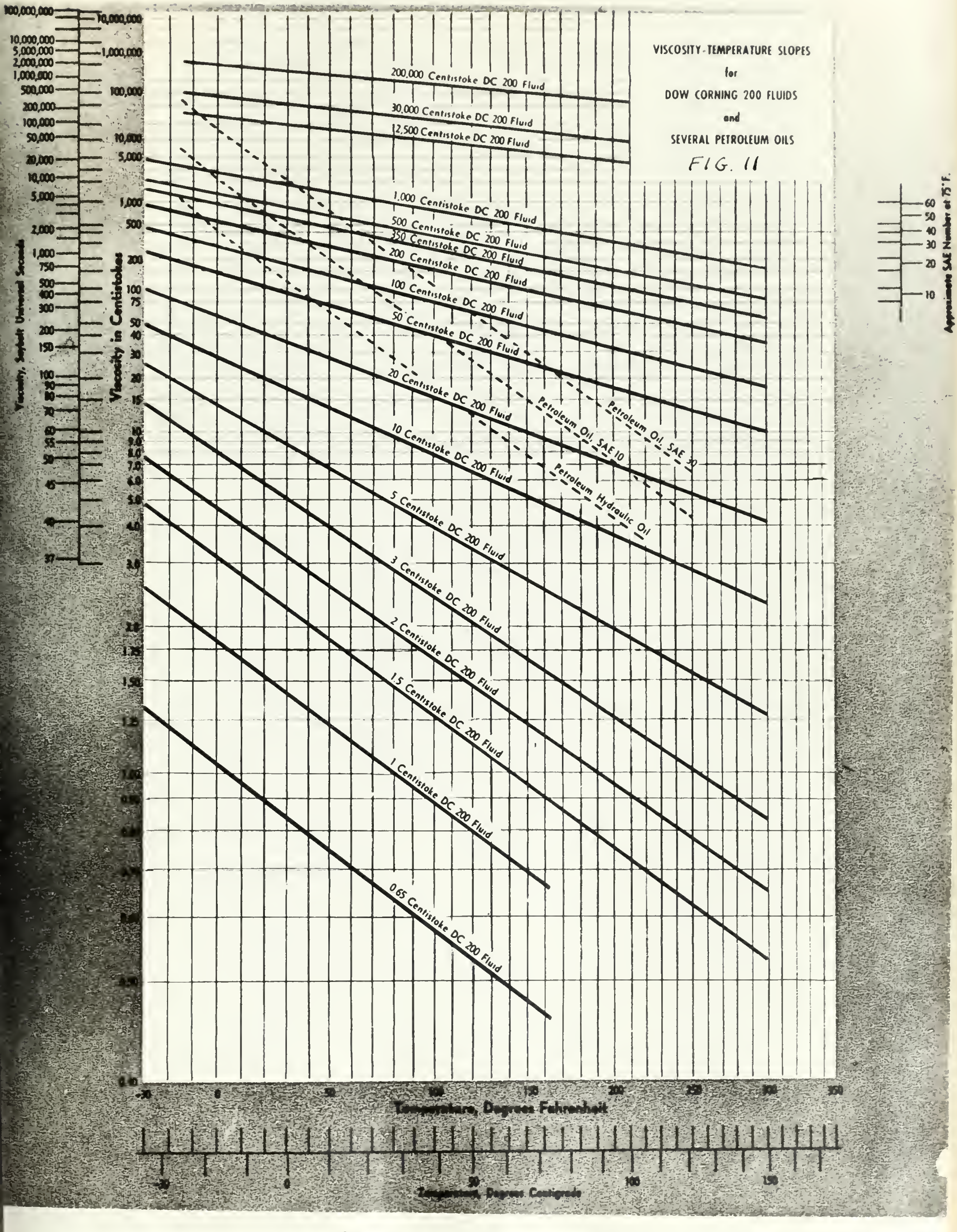
The two springs were manufactured from a chromium vanadium steel. They were annealed at a temperature of 1650 degrees Farenheit and tempered at 700 degrees Farenheit. A Rockwell "C" Hardness of 49 was obtained. The effective length of the cantilever spring is 1.84 inches.

When applied to an engine, the absorber will rotate uniformly with the engine mass system at the designed engine speeds because of the high viscosity of the fluid. When the engine passes through a critical speed which causes a torsional vibration, the absorber will continue to rotate at a constant speed. This results in a relative motion between the absorber and the housing which is amplified by the tuning of the springs. Consequently, energy is dissipated and the amplitude of the torsional oscillation is reduced.

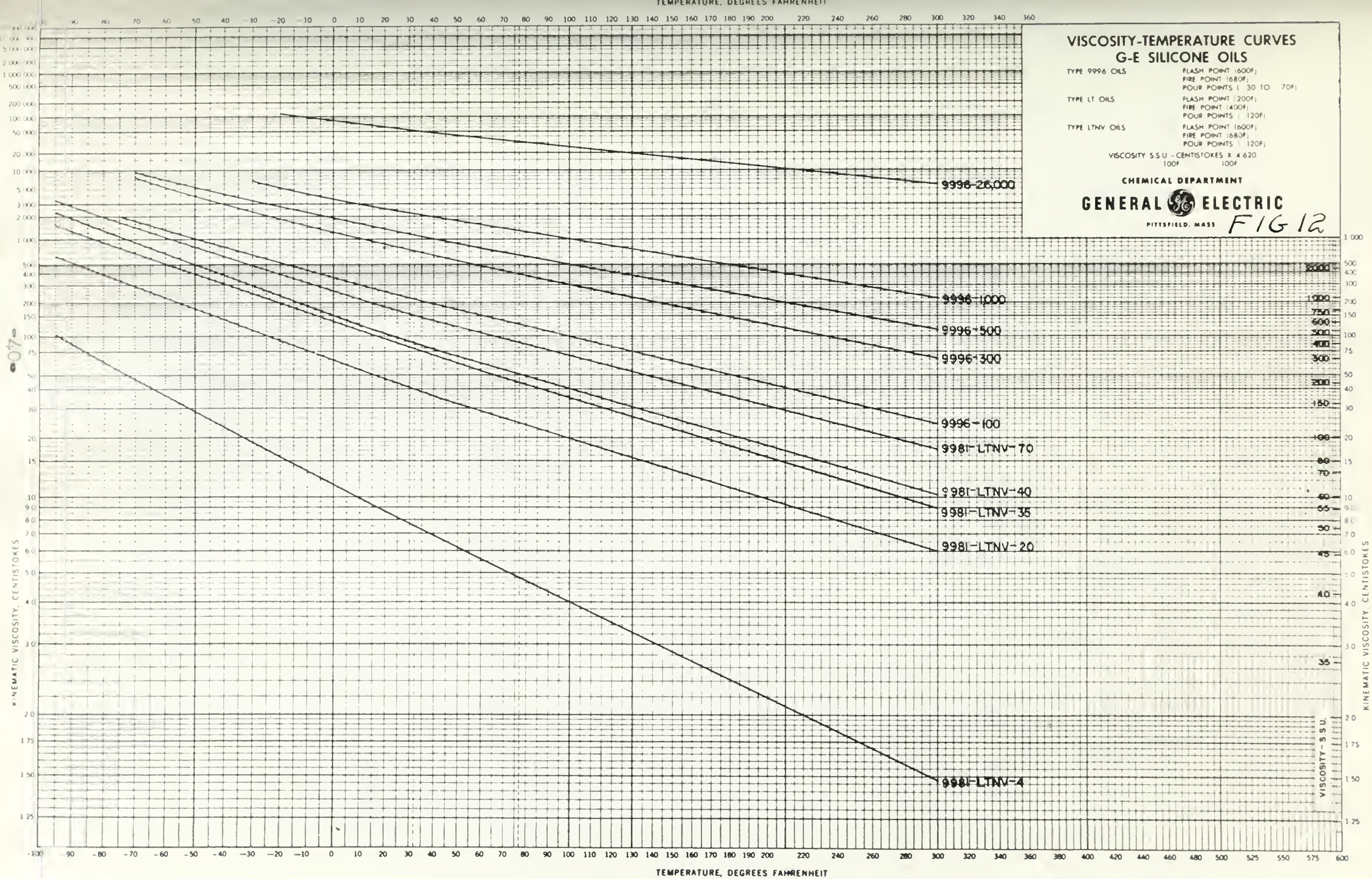
6. Silicone Oils.

Silicone oil was selected as the damping medium because of its very stable viscosity-temperature behaviour as is shown in figures (11) and (12). Also, the silicone oil has superior shear and oxidation resistance. It is commercially available in a wide range of viscosities. The following types of oils manufactured by the General Electric Company and the Dow Corning Corporation were used:

Manufacturer's Identification	Viscosity at Room Temp. 70 degrees Farenheit
GE 9996-1000	1300 centistokes (=5%)
GE 9996-500	640 "



VISCOSITY-TEMPERATURE SLOPES
for
DOW CORNING 200 FLUIDS
and
SEVERAL PETROLEUM OILS
FIG. 11
200,000 Centistoke DC 200 Fluid
30,000 Centistoke DC 200 Fluid
12,500 Centistoke DC 200 Fluid
1,000 Centistoke DC 200 Fluid
500 Centistoke DC 200 Fluid
350 Centistoke DC 200 Fluid
200 Centistoke DC 200 Fluid
100 Centistoke DC 200 Fluid
50 Centistoke DC 200 Fluid
20 Centistoke DC 200 Fluid
10 Centistoke DC 200 Fluid
5 Centistoke DC 200 Fluid
3 Centistoke DC 200 Fluid
2 Centistoke DC 200 Fluid
1.5 Centistoke DC 200 Fluid
1 Centistoke DC 200 Fluid
0.65 Centistoke DC 200 Fluid
Petroleum Oil, SAE 30
Petroleum Oil, SAE 10
Petroleum Hydraulic Oil
Viscosity, Saybolt Universal Seconds
Viscosity in Centistokes
Approximate SAE Number at 75° F.
Temperature, Degrees Fahrenheit
Temperature, Degrees Centigrade


VISCOSITY-TEMPERATURE CURVES
G-E SILICONE OILS
TYPE 9996 OILS
FLASH POINT (600F)
FIRE POINT (680F)
POUR POINTS (-30 TO -70F)
TYPE LT OILS
FLASH POINT (200F)
FIRE POINT (400F)
POUR POINTS -120F)
TYPE LTNV OILS
FLASH POINT (600F)
FIRE POINT (680F)
POUR POINTS -120F)
VISCOSITY S.S.U. - CENTISTOKES X 4.620
100F 100F
CHEMICAL DEPARTMENT
GENERAL ELECTRIC
PITTSFIELD, MASS
FIG 12
9996-26,000
9996-1,000
9996-500
9996-300
9996-100
9981-LTNV-70
9981-LTNV-40
9981-LTNV-35
9981-LTNV-20
9981-LTNV-4
TEMPERATURE, DEGREES FAHRENHEIT
KINEMATIC VISCOSITY, CENTISTOKES
VISCOSITY - S.S.U.

GE 9996-300	400 centistokes		(±5%)
GE 9996-100	125	"	"
DC 200 -1000	1000 centistokes		"
DC 200 -500	500	"	"
DC 200 -350	350	"	"
DC 200 -200	200	"	"
DC 200 -100	100	"	"
DC 200 -50	50	"	"

One of the undesirable properties of a silicone oil is its poor oiliness characteristics under thin film conditions between certain combinations of metals. This is particularly true for the combination of steel on steel. For this reason it was necessary to cadmium plate the absorber mass.

7. Calibration of Main Shaft.-

The main shaft was calibrated by determining the torsional spring constant for various positions of the fixed end pedestal along the guide plate. A pointer mounted on the base of the pedestal slides over a meter stick which is secured to the guide plate. Each position of the fixed end pedestal is identified by the position number indicated by the pointer. At various positions scaled weights were attached to the driving arm and the deflections measured by the traveling microscope. In Fig. 13 to 26 the total weight added to the driving arm is plotted against the deflection. From the slopes of these curves the specific torsional spring constants for given positions of the pointer were obtained. These values are plotted against the position number in Fig. 27.

8. Calibration of Absorber Shaft.-

To calibrate the absorber shaft, the part of the shaft to which the absorber shaft is keyed was held rigidly by the fixed end pedestal.

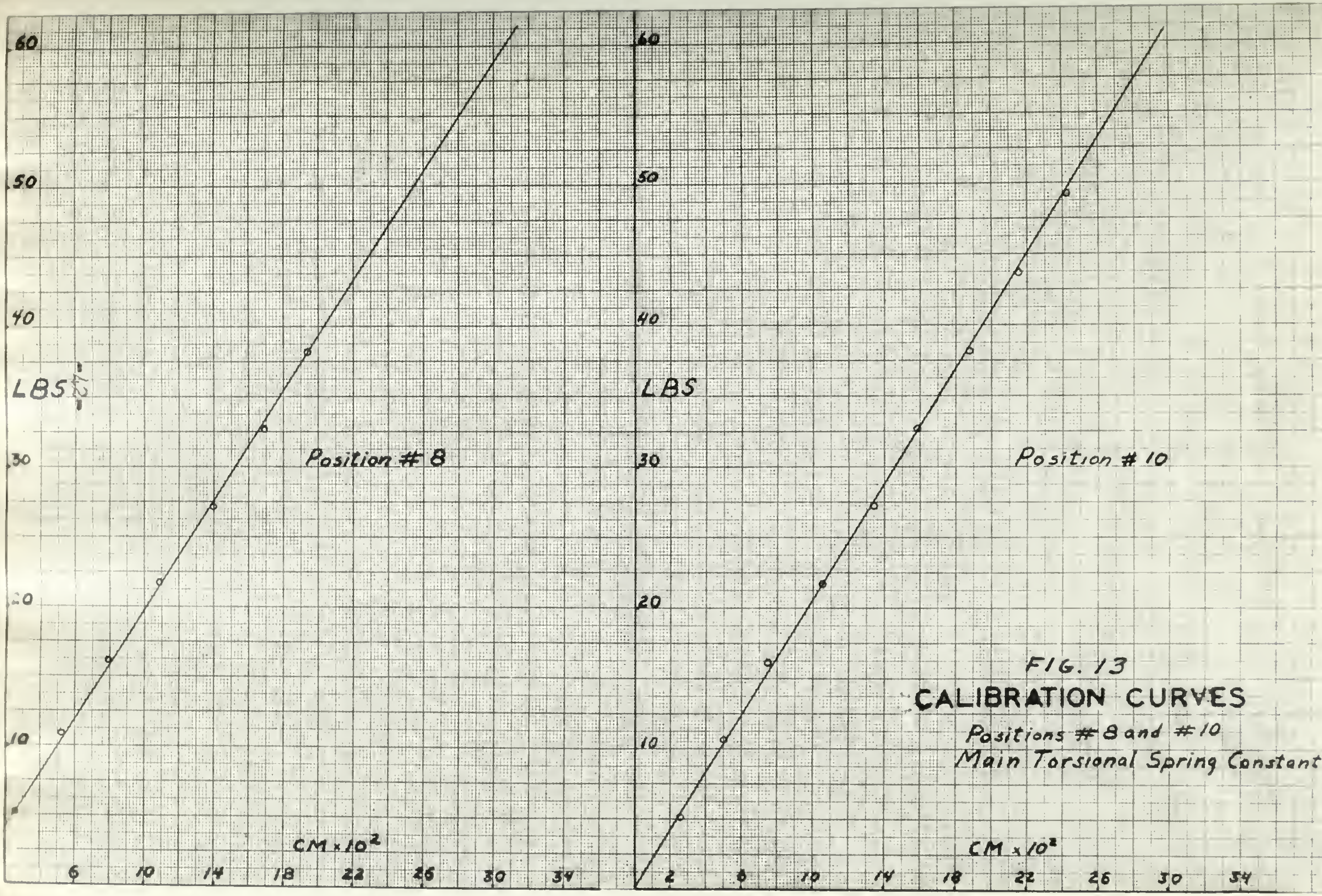


FIG. 13
CALIBRATION CURVES
Positions # 8 and # 10
Main Torsional Spring Constant

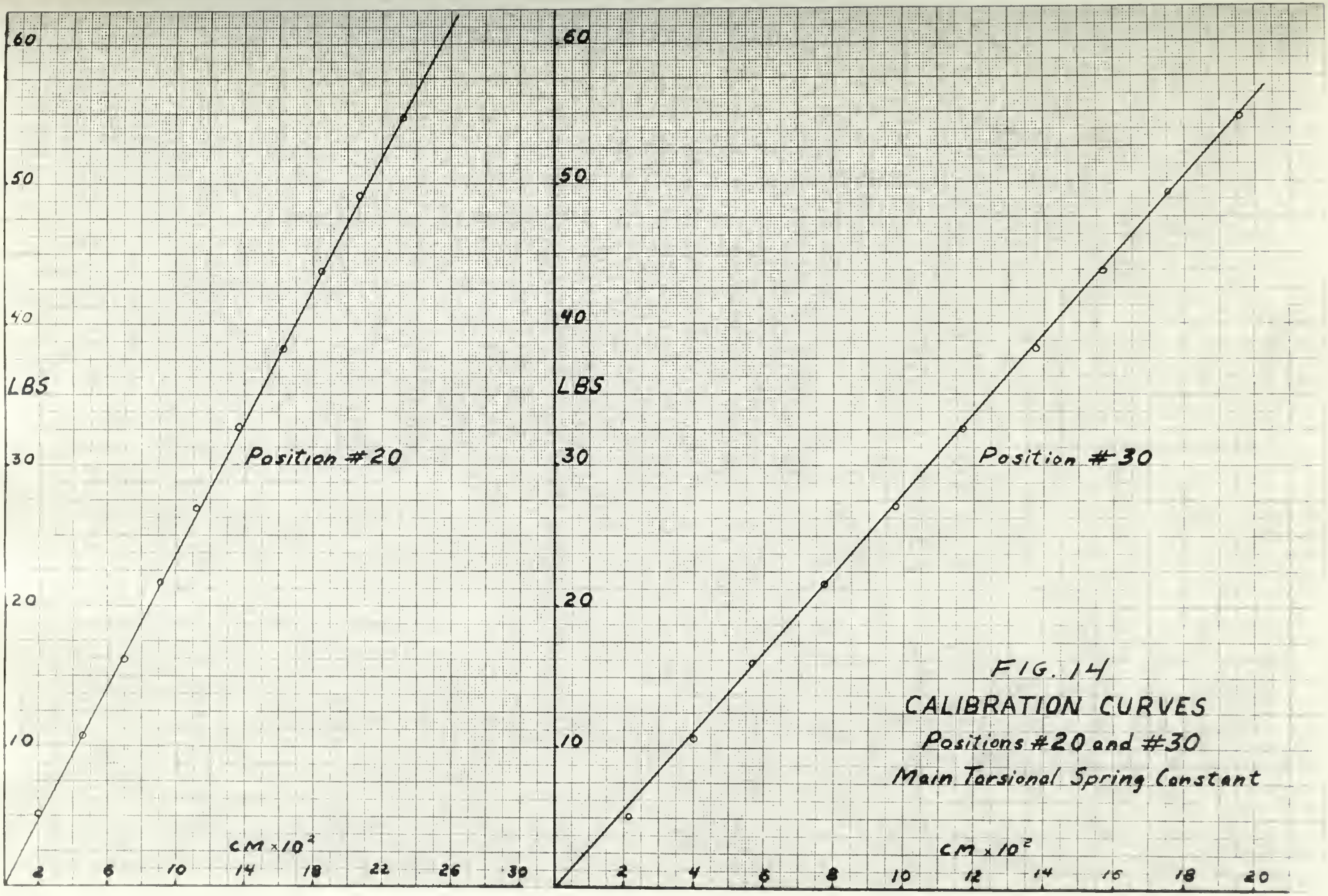


FIG. 14
CALIBRATION CURVES
Positions #20 and #30
Main Torsional Spring Constant

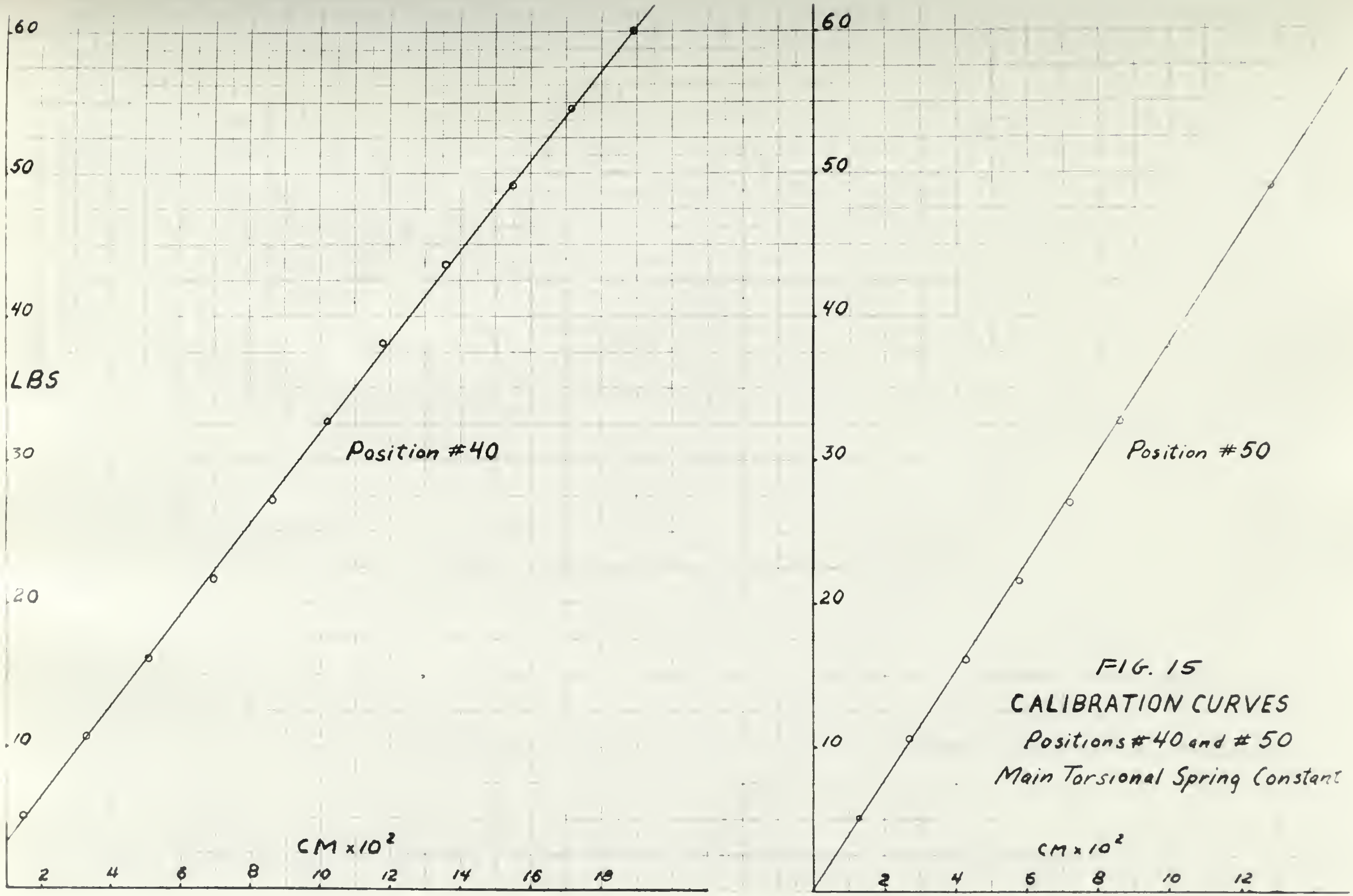


FIG. 15
CALIBRATION CURVES
Positions #40 and #50
Main Torsional Spring Constant

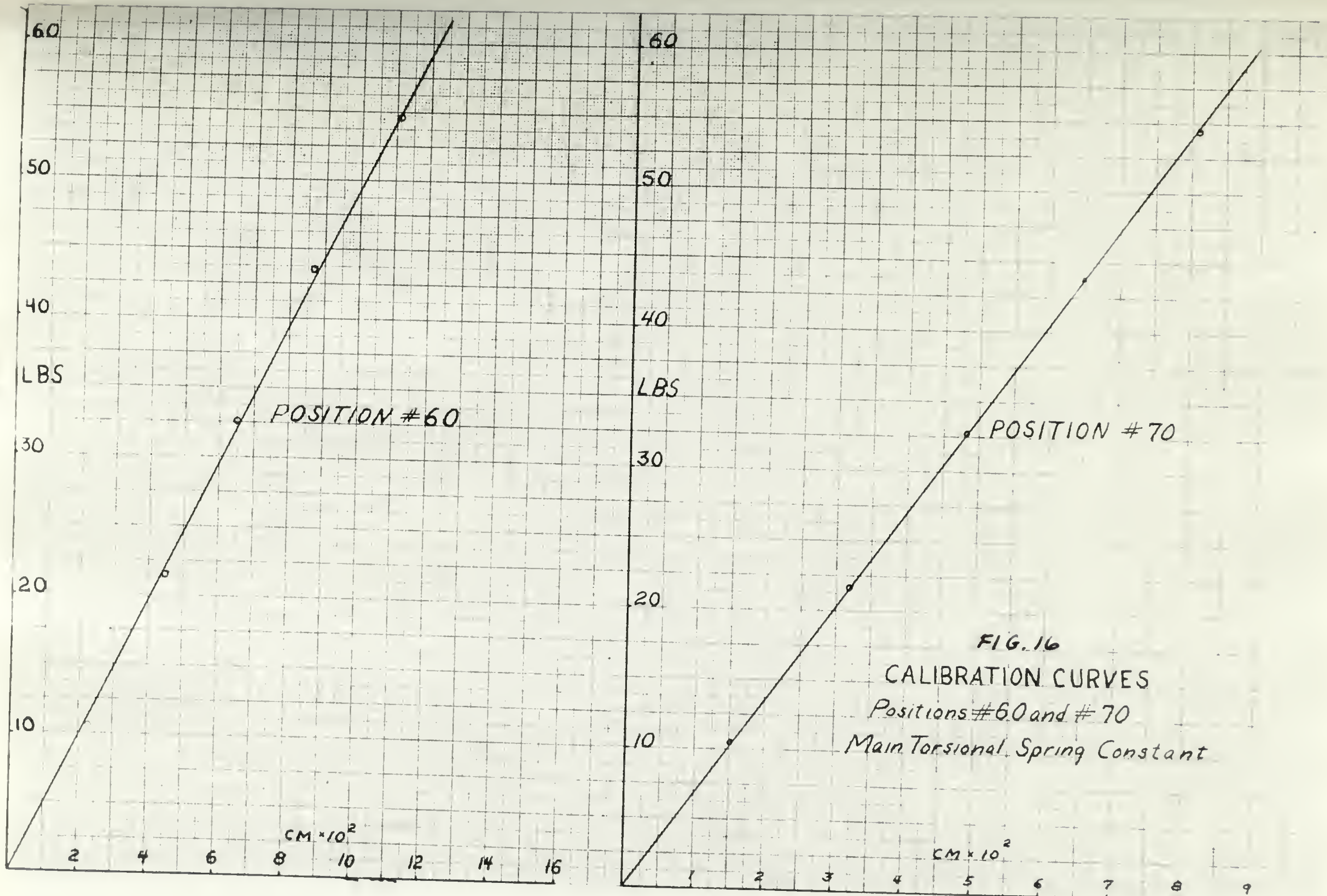


FIG. 16
CALIBRATION CURVES
Positions #60 and #70
Main Torsional Spring Constant

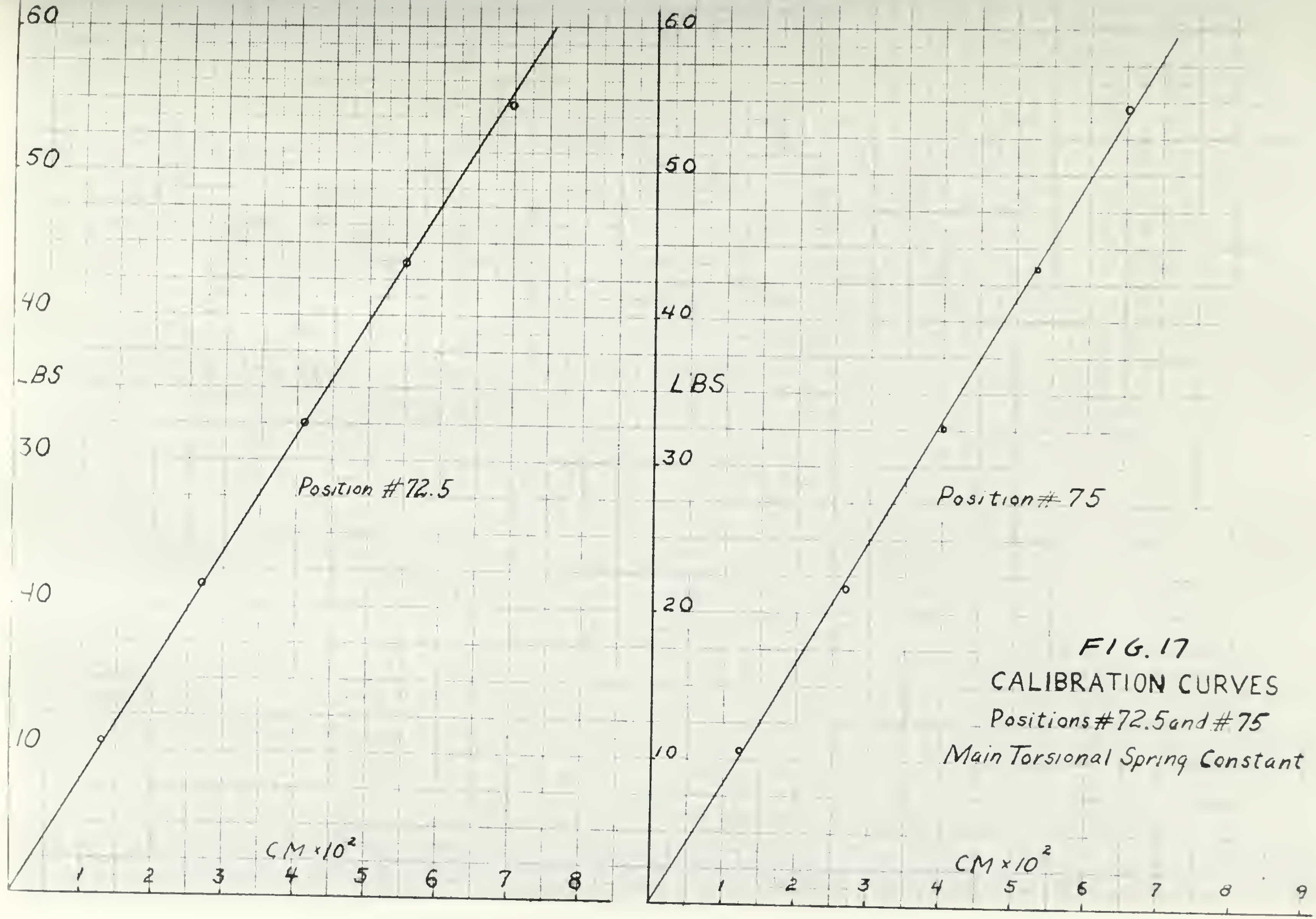


FIG. 17
CALIBRATION CURVES
Positions #72.5 and #75
Main Torsional Spring Constant

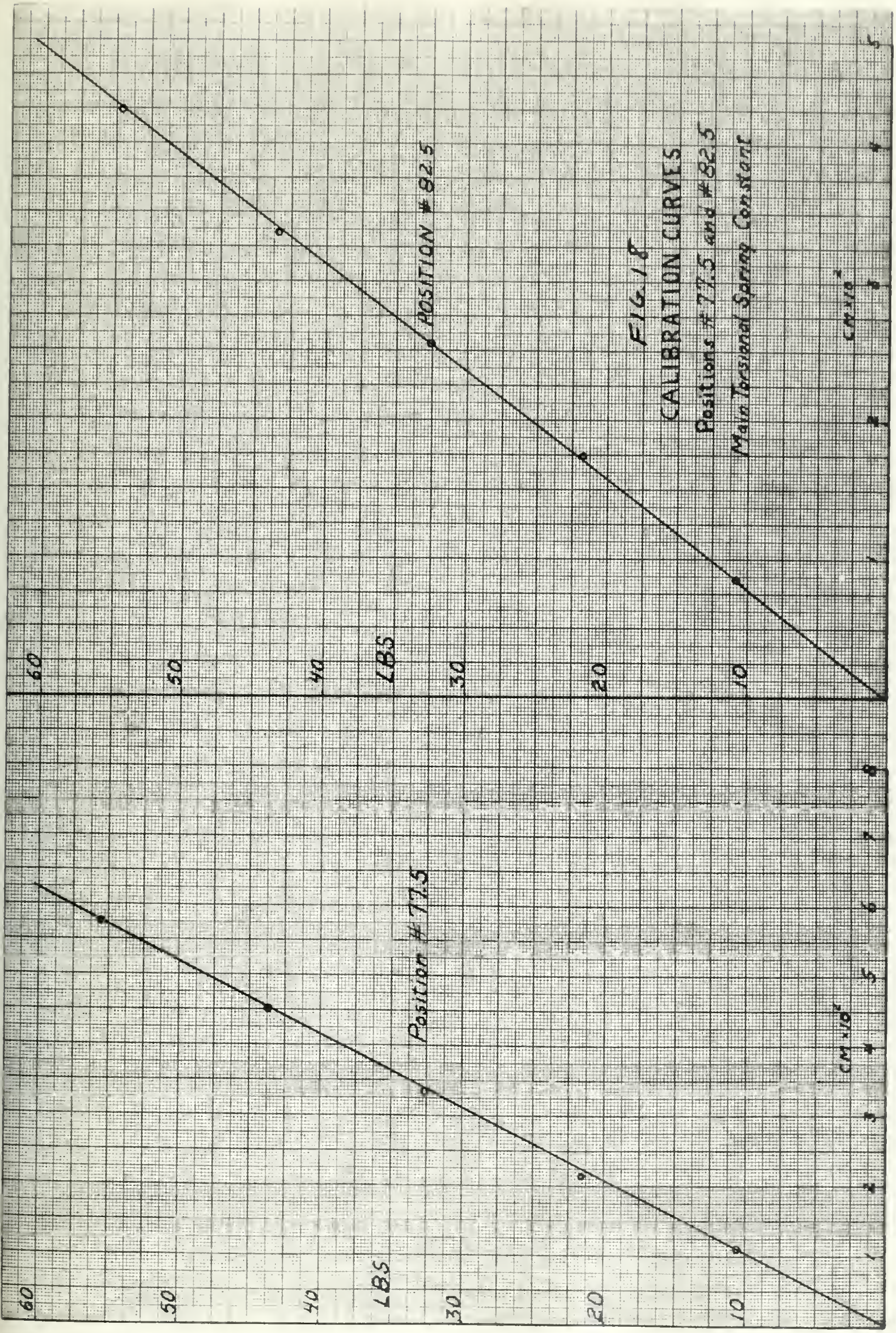


FIG. 18
CALIBRATION CURVES
Positions #77.5 and #82.5
Main Torsional Spring Constant

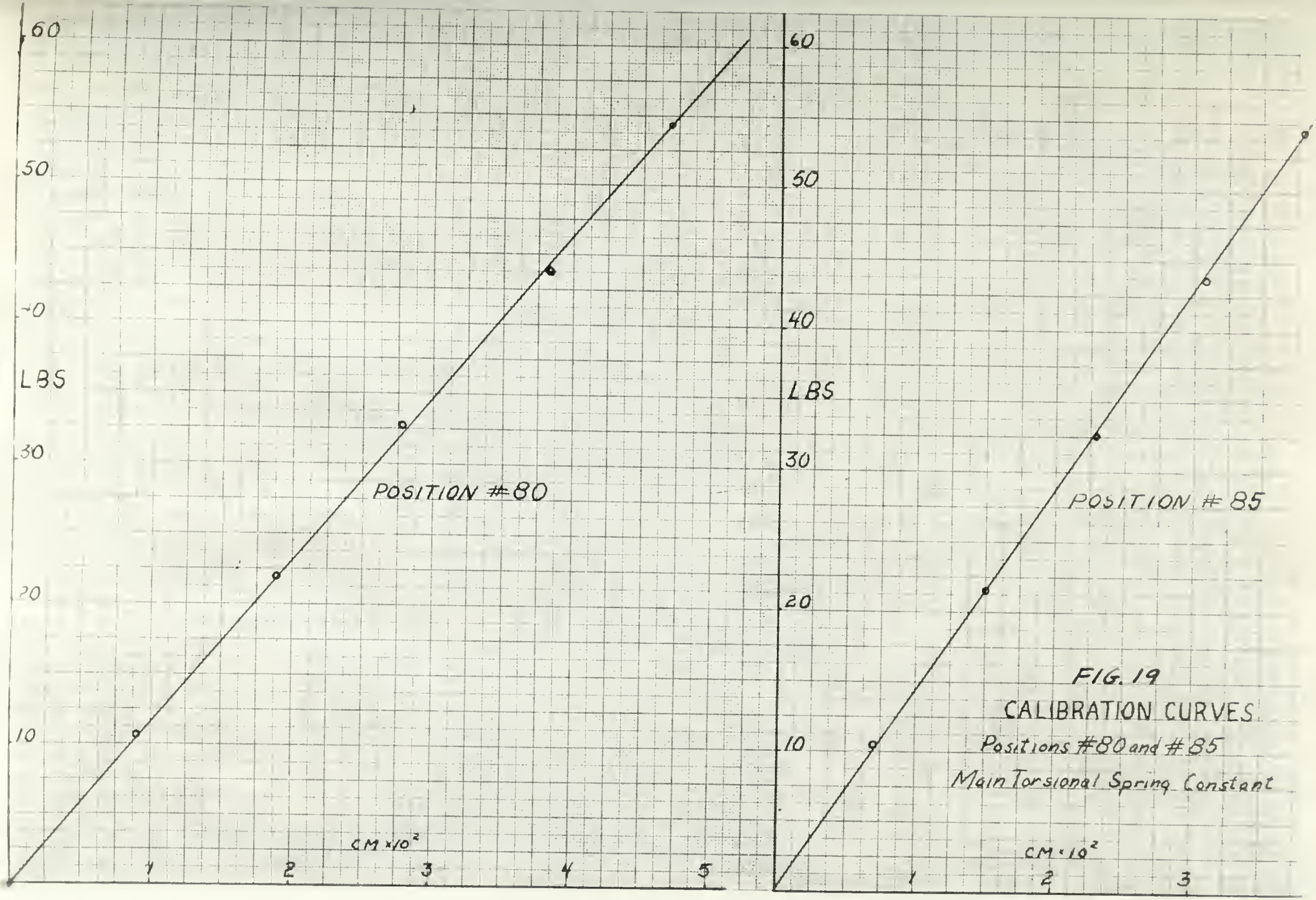


FIG. 19
CALIBRATION CURVES
Positions #80 and #85
Main Torsional Spring Constant

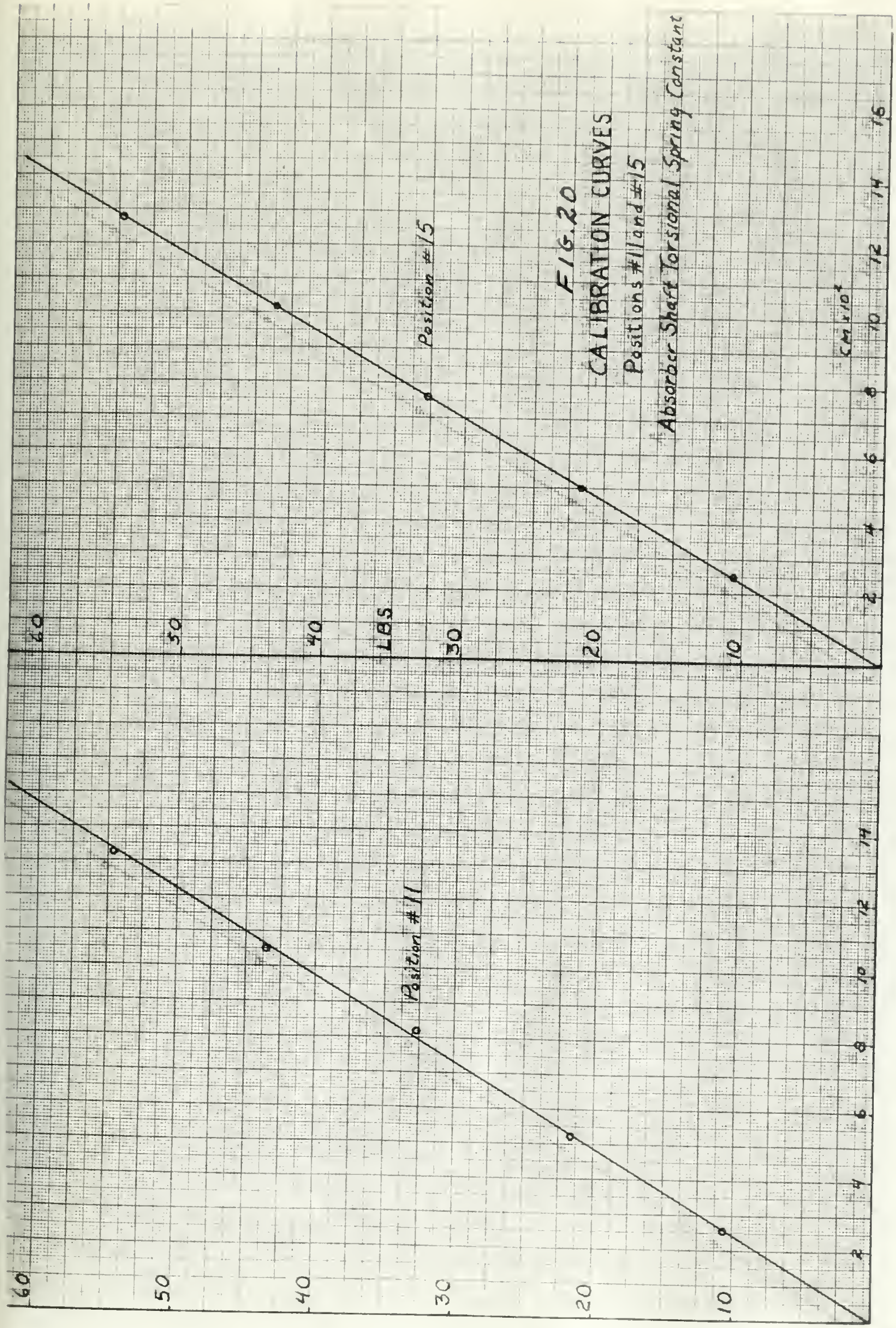


FIG. 20
CALIBRATION CURVES
Positions #11 and #15
Absorber Shaft Torsional Spring Constant
Position #15
Position #11
LBS
cm x 10²
10
20
30
40
50
60
2
4
6
8
10
12
14
16

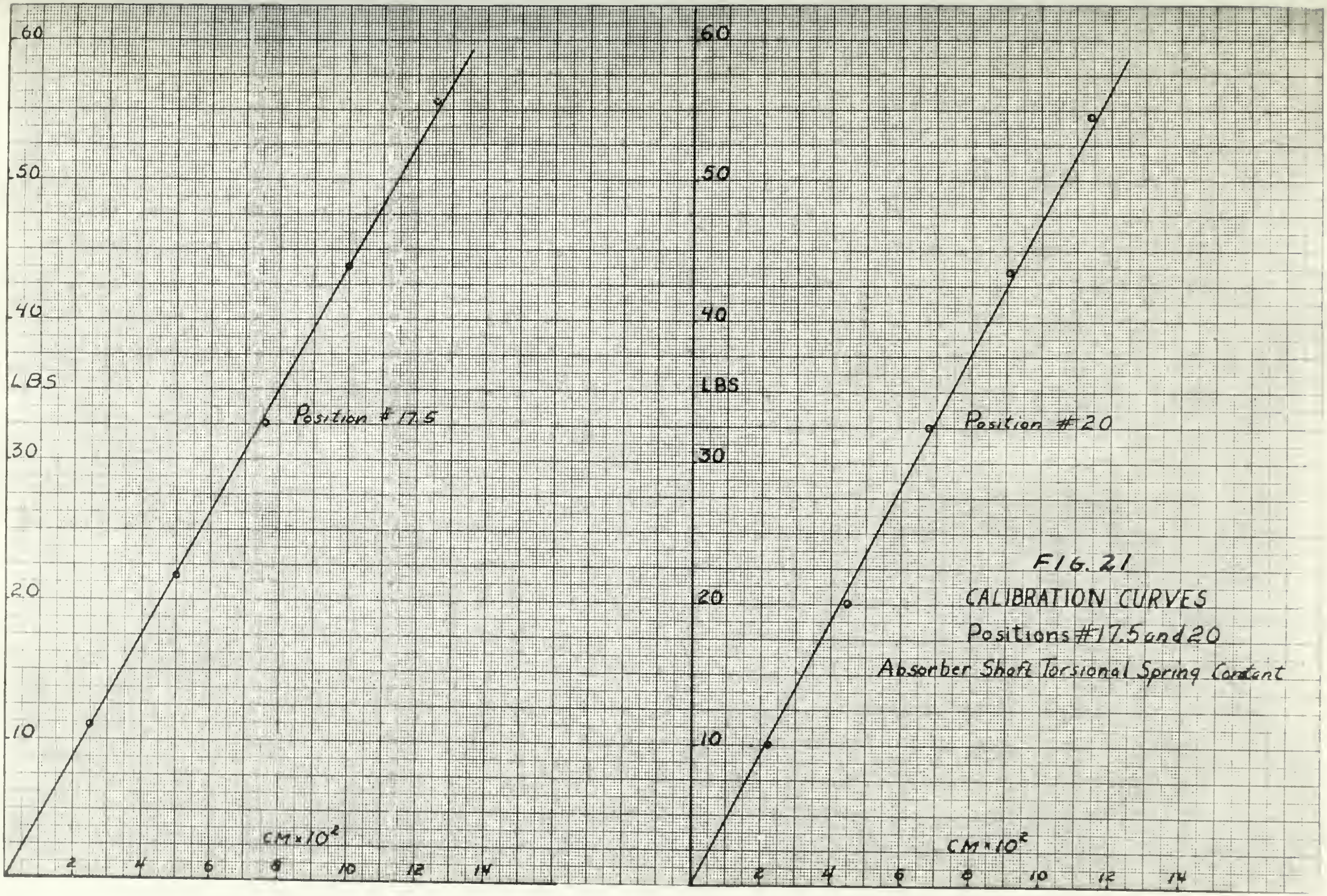


FIG. 21
CALIBRATION CURVES
Positions #17.5 and 20
Absorber Shaft Torsional Spring Constant

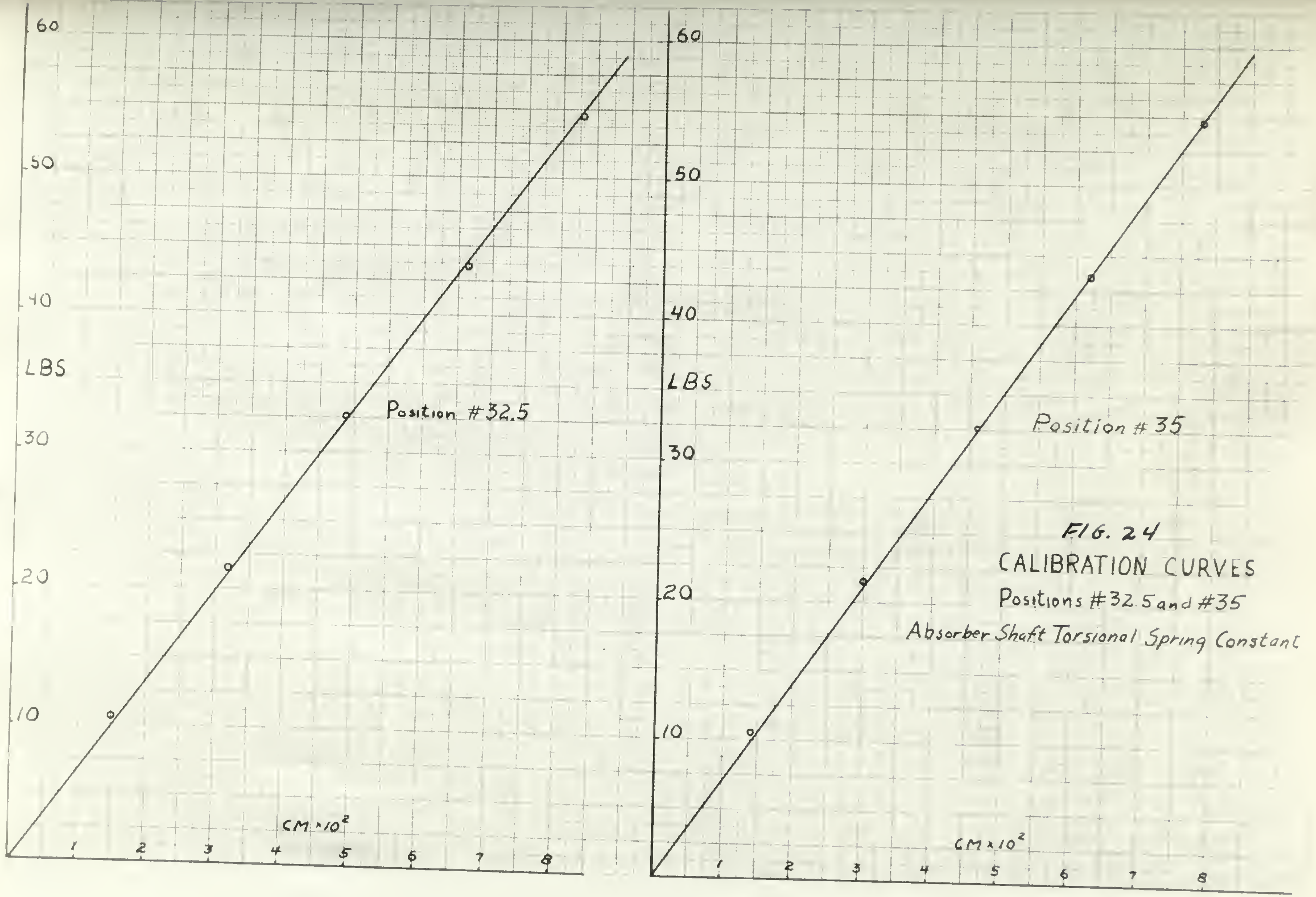


FIG. 24
CALIBRATION CURVES
Positions #32.5 and #35
Absorber Shaft Torsional Spring Constant

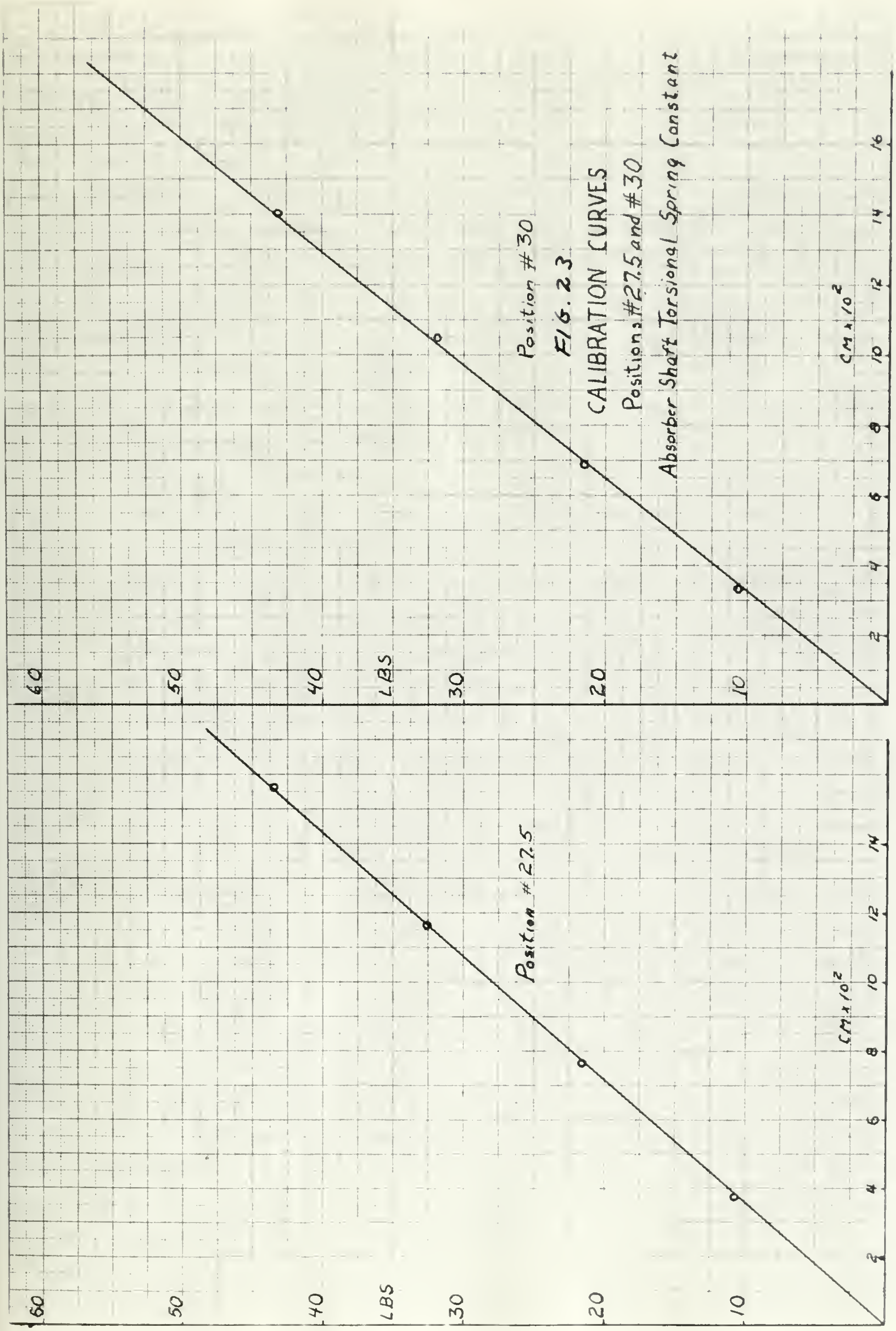


FIG. 23
CALIBRATION CURVES
Positions #27.5 and #30
Absorber Shaft Torsional Spring Constant

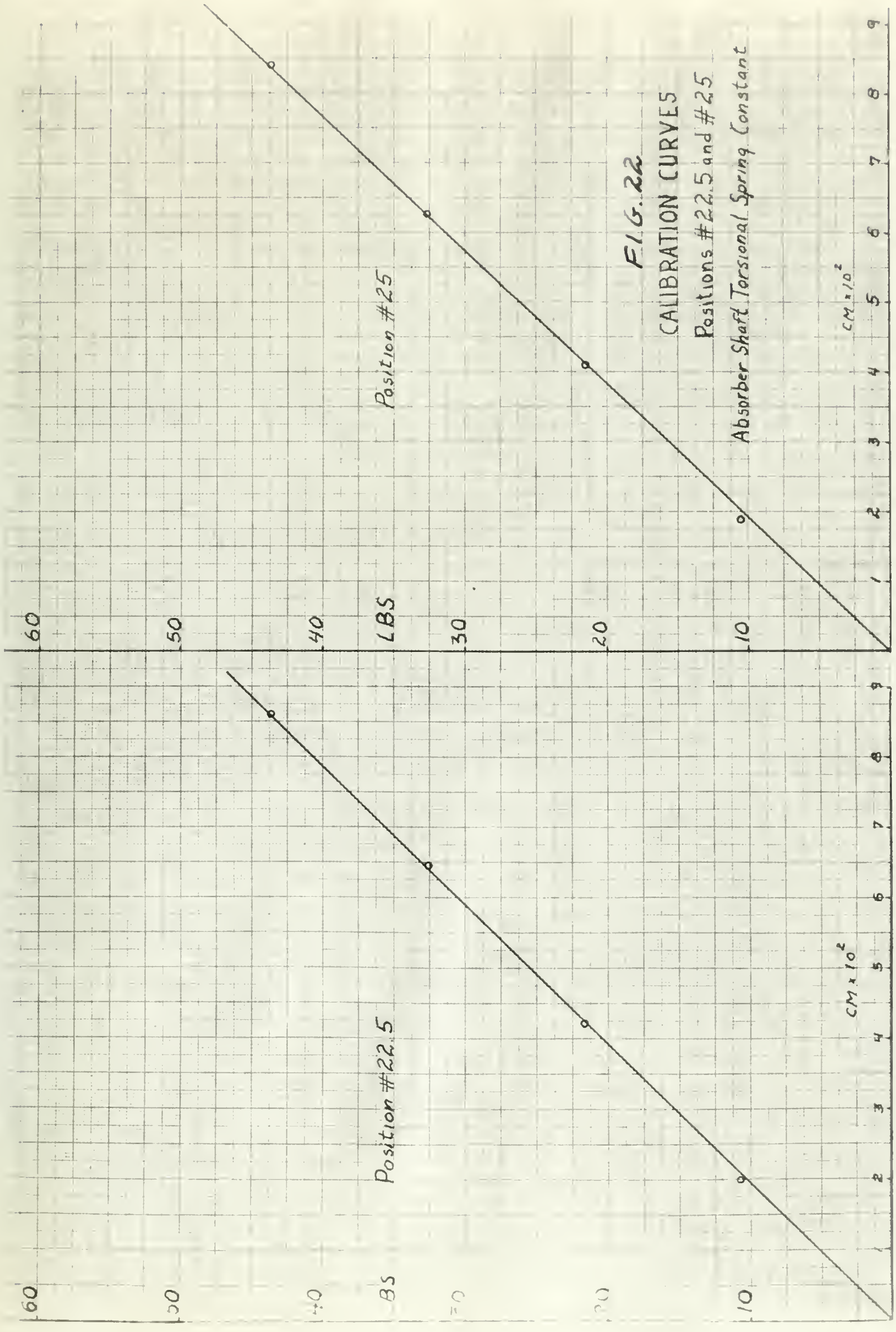


FIG. 22
CALIBRATION CURVES
Positions #22.5 and #25
Absorber Shaft Torsional Spring Constant

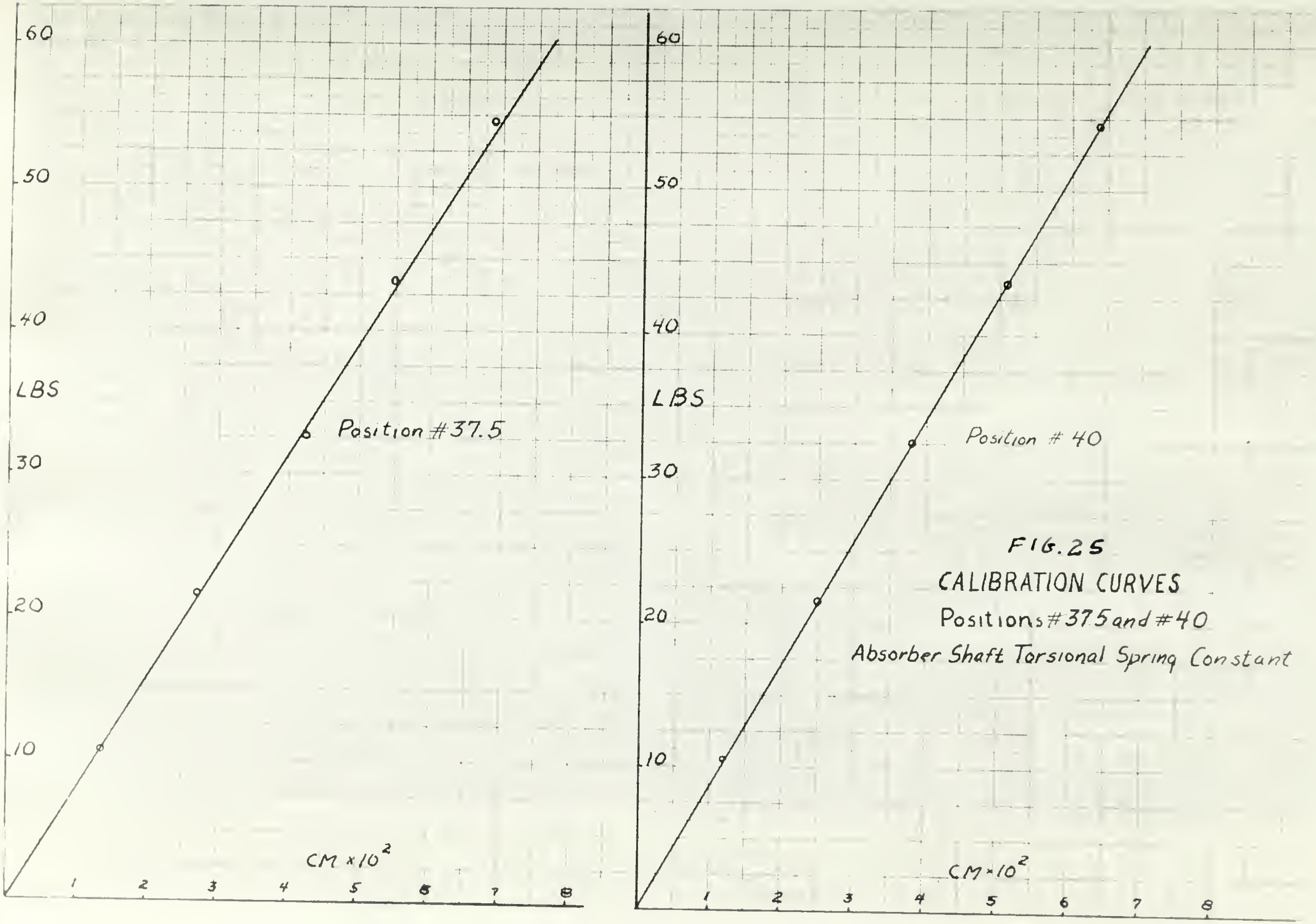


FIG. 25
CALIBRATION CURVES
Positions #37.5 and #40
Absorber Shaft Torsional Spring Constant

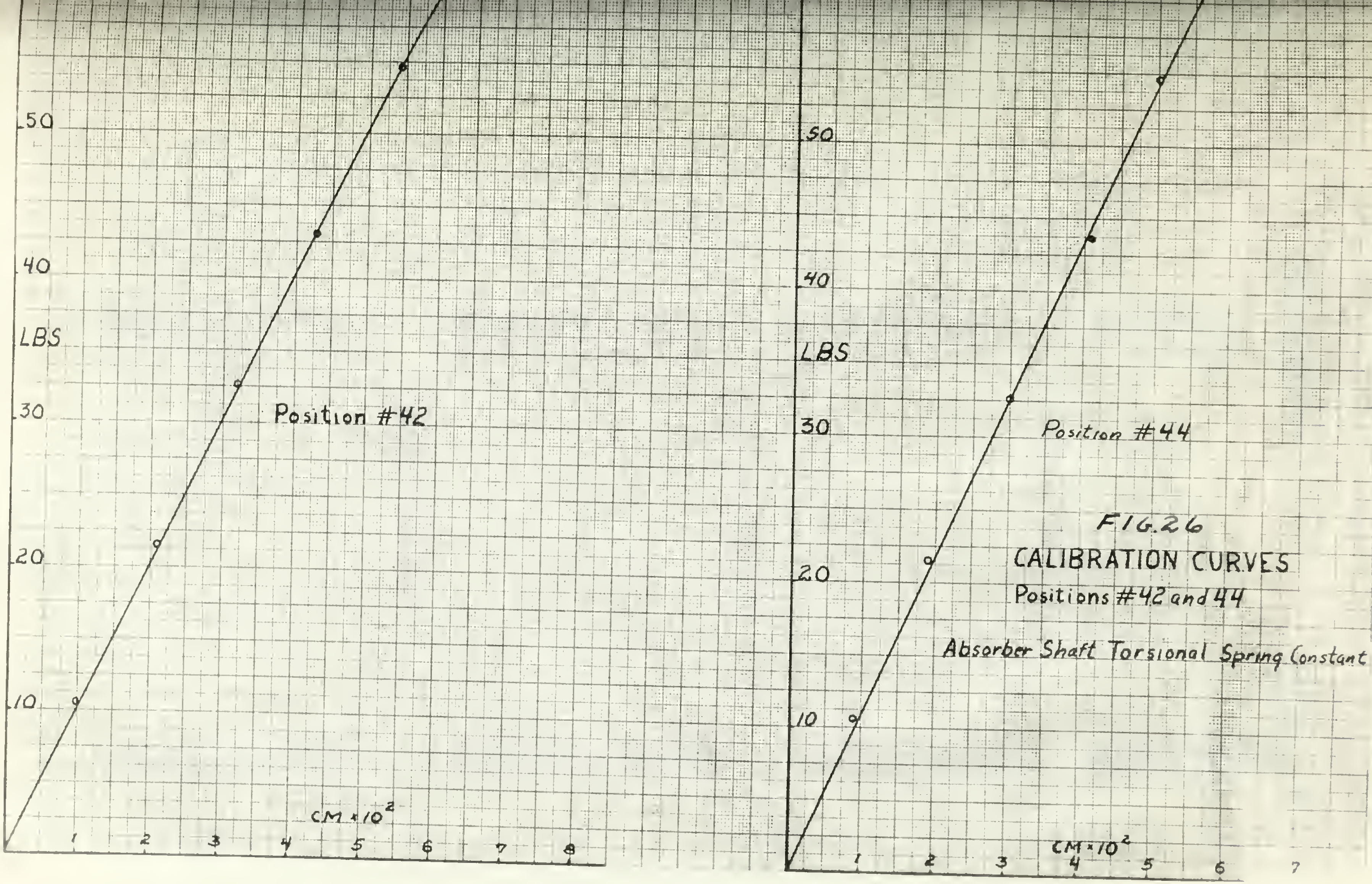


FIG. 26
CALIBRATION CURVES
Positions #42 and 44

Absorber Shaft Torsional Spring Constant

TABLE 1

Calibration Data for Main Torsional Spring Constant

Position Number 8

Weight added (lbs)	Deflection (Cm)
5.2	.026
10.7	.052
16.2	.079
21.7	.109
27.2	.140
32.7	.169
38.2	.194

Position Number 10

5.2	.026
10.7	.050
16.2	.075
21.7	.105
27.2	.134
32.7	.159
38.2	.186
43.7	.216
49.2	.243

Position Number 20

5.2	.020
10.7	.046
16.2	.070
21.7	.091
27.2	.112
32.7	.137
38.2	.163
43.7	.185
49.2	.207
54.7	.233
60.2	.261

Position Number 30

5.1	.022
10.6	.040
16.1	.057
21.6	.077
27.1	.098
32.6	.119
38.1	.138
43.6	.157
49.1	.175
54.6	.196
60.1	.221

Weight added (lbs)	Deflection (Cm)
Position Number 40	
5.1	.015
10.6	.033
16.1	.051
21.6	.070
27.1	.087
32.6	.102
38.1	.118
43.6	.136
49.1	.155
54.6	.172
60.1	.189
Position Number 50	
5.1	.013
10.6	.027
16.1	.043
21.6	.058
27.1	.072
32.6	.086
38.1	.099
43.6	.113
49.1	.128
54.6	.143
60.1	.158
Position Number 60	
10.6	.023
21.6	.045
32.6	.065
43.6	.087
54.6	.111
65.6	.135
Position Number 70	
10.6	.015
21.6	.032
32.6	.048
43.6	.065
54.6	.081
65.6	.095

Weight added (lbs)	Deflection (Cm)
Position Number 72.5	
10.6	.013
21.6	.027
32.6	.041
43.6	.055
54.6	.070
65.6	.086
Position Number 75	
10.6	.012
21.6	.027
32.6	.040
43.6	.053
54.6	.065
65.6	.078
Position Number 77.5	
10.6	.011
21.6	.021
32.6	.033
43.6	.045
54.6	.058
65.6	.070
Position Number 80	
10.6	.009
21.6	.019
32.6	.028
43.6	.036
54.6	.047
65.6	.057
Position Number 82.5	
10.6	.011
21.6	.020
32.6	.028
43.6	.036
54.6	.045
65.6	.052

Weight added (lbs)	Deflection (Cm)
Position Number 85	
10.6	.007
21.6	.015
32.6	.023
43.6	.031
54.6	.038
65.6	.045

TABLE II

Calibration Data for Absorber Shaft Torsional Spring Constant

Weight added (lbs)	Deflection (Cm)
Position Number 11	
10.6	.023
21.6	.051
32.6	.076
43.6	.105
54.6	.132
65.6	.158
Position Number 15	
10.6	.024
21.6	.050
32.6	.076
43.6	.082
54.6	.108
Position Number 17.5	
10.6	.025
21.6	.050
32.6	.075
43.6	.100
54.6	.125
Position Number 20	
10.6	.022
21.6	.045
32.6	.068
43.6	.091
54.6	.114

Weight added (lbs)	Deflection (Cm)
Position Number 22.5	
10.6	.020
21.6	.042
32.6	.064
43.6	.086
54.6	.108
Position Number 25	
10.6	.019
21.6	.041
32.6	.063
43.6	.084
54.6	.105
Position Number 27.5	
10.6	.018
21.6	.038
32.6	.058
43.6	.078
54.6	.098
Position Number 30	
10.6	.016
21.6	.034
32.6	.052
43.6	.070
54.6	.093
Position Number 32.5	
10.6	.015
21.6	.032
32.6	.049
43.6	.066
54.6	.083
Position Number 35	
10.6	.014
21.6	.030
32.6	.046
43.6	.062
54.6	.078

Weight added (lbs)	Deflection (Cm)
Position Number 37.5	
10.6	.013
21.6	.027
32.6	.041
43.6	.055
54.6	.069
Position Number 40	
10.6	.012
21.6	.025
32.6	.038
43.6	.051
54.6	.064
Position Number 42	
10.6	.012
21.6	.025
32.6	.038
43.6	.051
54.6	.064
Position Number 44	
10.6	.009
21.6	.019
32.6	.030
43.6	.041
54.6	.050

Position Number	Main Torsional Spring Constant (Lb-in per radian)
8	12,500
10	13,000
20	15,000
30	17,800
40	20,100
50	24,300
60	31,200
70	42,800
72.5	47,300
75	52,200
77.5	61,000
80	73,000
82.5	77,500
85	91,400

Position Number	Absorber Shaft Torsional Spring Constant(lb-in per radian)
11	26,500
15	27,200
17.5	28,000
20	29,800
22.5	32,300
25	33,200
27.5	35,900
30	39,400
32.5	42,000
35	44,500
37.5	50,000
40	54,500
42	64,000
44	68,200

Experimental Data for Determination of Absorber Torsional Spring Constant.

Weight Added (lbs)	Deflection (Cm)
5.6	.020
11.1	.040
16.6	.060
22.1	.079
27.6	.096

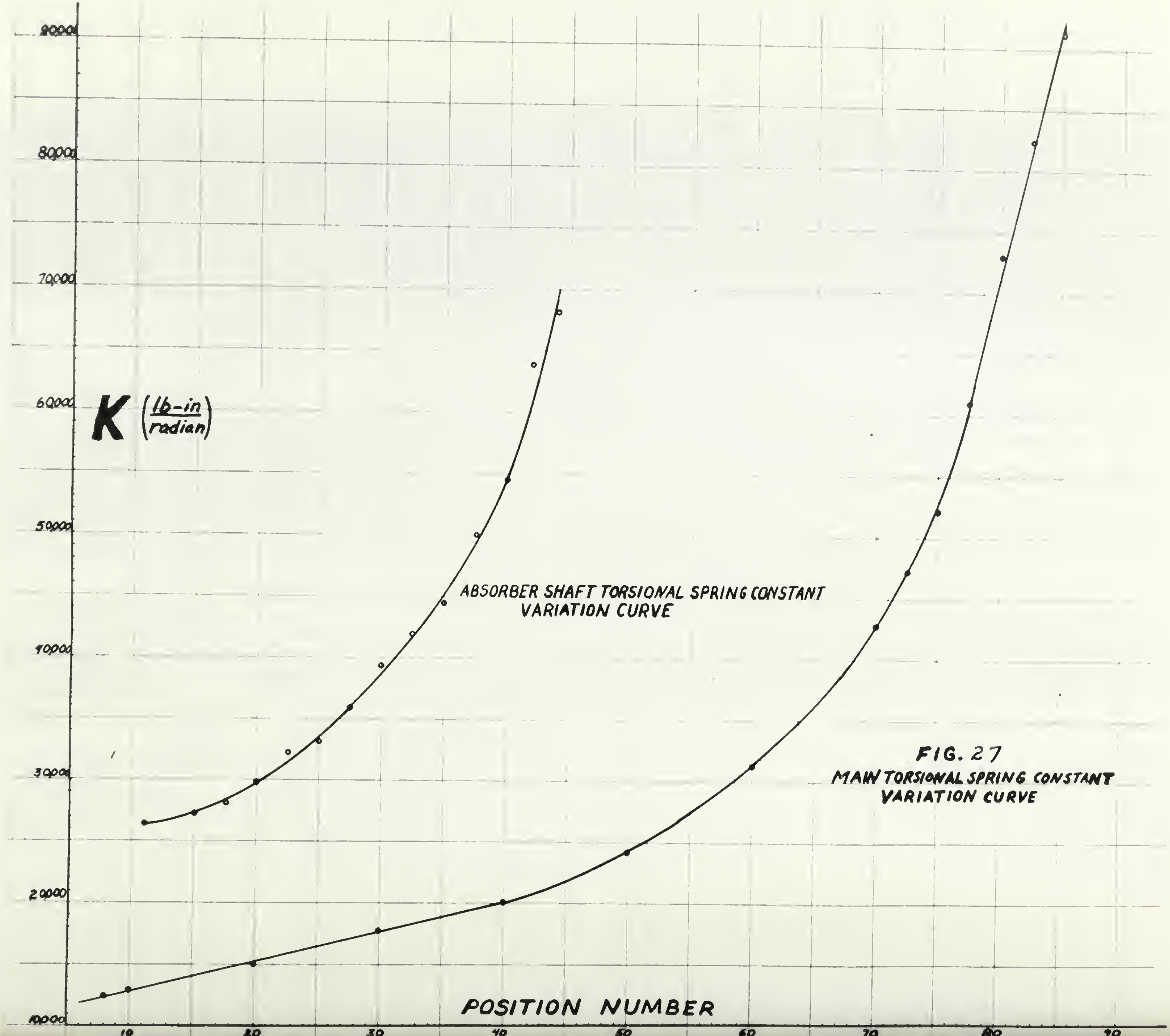
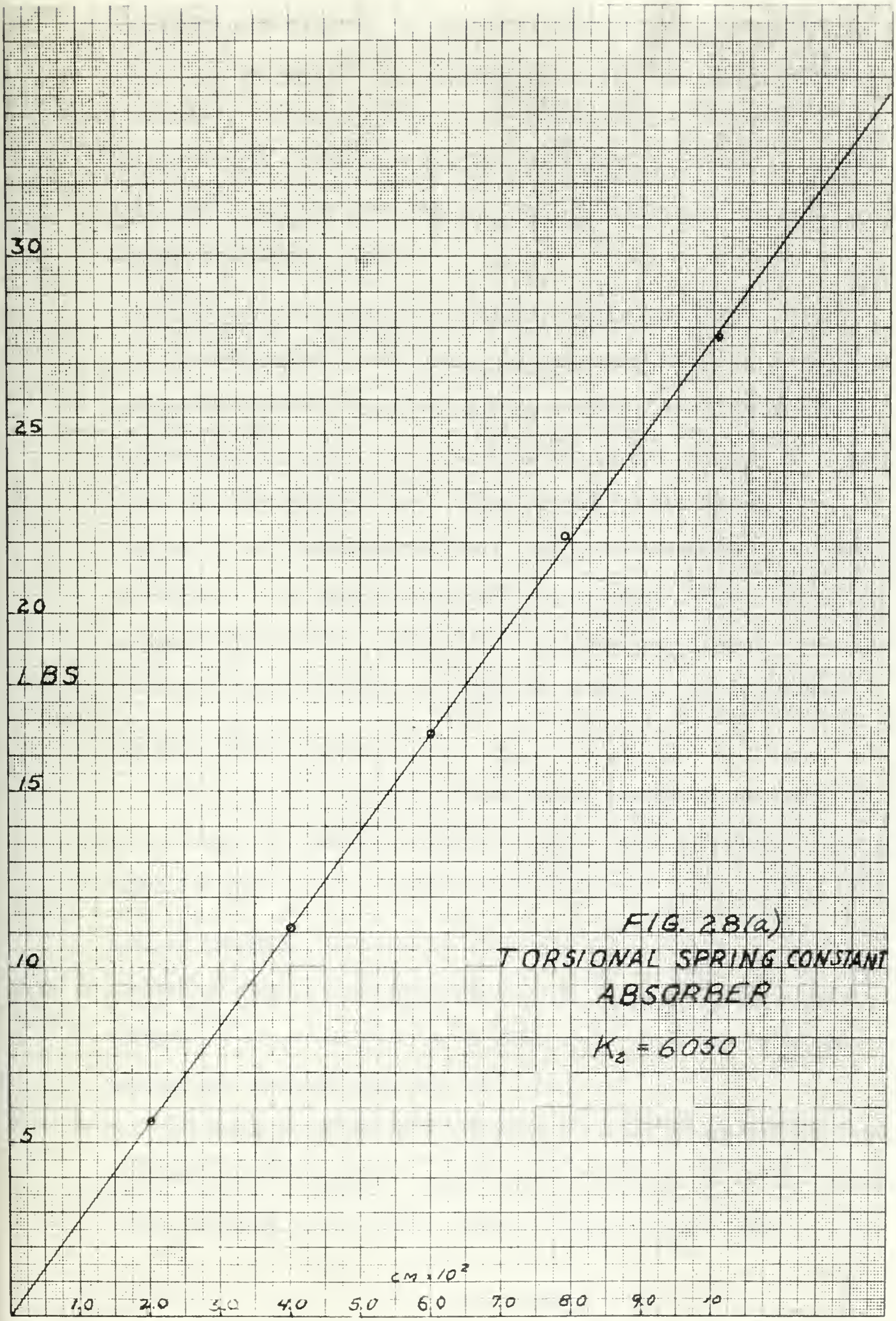


FIG. 27
MAIN TORSIONAL SPRING CONSTANT
VARIATION CURVE


30
25
20
LBS
15
10
5
FIG. 28(a)
TORSIONAL SPRING CONSTANT
ABSORBER
$K_2 = 6050$
$CM \times 10^2$
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10

At various positions of the pedestal scaled weights were attached to the driving arm and the deflections measured by the traveling microscope. From the slopes of the weight versus deflection the absorber shaft torsional spring constant were determined as for the main shaft. The absorber shaft torsional spring constant is plotted against position in Fig. 27.

9. Determination of Absorber Torsional Spring Constant.-

The equivalent torsional spring constant of the two cantilever springs on which the absorber mass is suspended was determined by placing scaled weights on a tray attached to the absorber mass and measuring the statical linear displacements by the traveling microscope. The experimental data for this determination is presented in Table I . In Fig. 28 the total weight added to the driving arm is plotted against the deflection. From the slop of this curve the torsional spring constant was found to be 6050 lb-in per radian.

10. Experimental Results.-

The first step of the experimental investigation consisted of determining the moment of inertia of the driving arm assembly and the vibration motor by use of the conventional formula, $\Omega = \sqrt{\frac{K}{I}}$. The natural frequency was found by observing the amplitude of the sinusoidal wave on the oscillograph. The value of K, the main torsional spring constant, for a given position of the fixed end support was obtained from Fig. 27.

For various values of K approximately equal values of the moment of inertia were obtained. This values were within 3% of the computed values based on the mass and geometry of the various parts. It was

observed while determining the natural frequency for various values of K that results could be repeated so long as the amplitude of the vibration was kept constant. If the amplitude control on the audio oscillator was changed, the natural frequency of the system changed and results could not be repeated. It was also observed that a considerable variation in the natural frequency of the system could be obtained by adjusting the two allen screws on the bushing pedestal. This, in effect, varies the damping in the system.

In order to keep these two factors constant throughout the experimental investigation, the amplitude control on audio oscillator was kept set on position number 20 and gain on the oscillograph was kept set on position number 16. For these two settings, a displacement of .016 centimeter as measured by the traveling microscope at the driving arms corresponded to a displacement of 20 divisions on the cathode ray tube screen. Likewise, to minimize the effect of any change in damping, no adjustment was made on the bushing pedestal during the course of the experiment.

The next step in the investigation consisted in determining the main moment of inertia by finding the natural frequency of the system with the absorber assembly mounted on the shaft. The values thus obtained for the moment of inertia could not be repeated and correlated with the simple relationship, $\Omega = \sqrt{\frac{K}{I}}$. It was observed that there was a perceptible transverse vibration present along the entire length of the shaft. The natural frequency could not be predicted by assuming a purely torsional vibration. The original assumption that the

vibration in the system would be predominantly torsional was proven to be incorrect. With the addition of the absorber assembly the vibration motor was causing the system to vibrate both torsionally and transversely. Therefore, no attempt was made to correlate the theory presented in Chapter II with the experimental analysis.

When it was determined that the tranverse forces were of appreciable magnitude, the investigation was directed at making a comparative study of the two types of absorbers under identical conditions of operation. Since in an actual engine application there is no such phenomena as a purely torsional vibration, a study of the operation of both types of vibration absorbers under the influence of combined tranverse and torsional forces would be of some significance.

The next procedure was to investigate the operation of the absorber over a range of natural frequencies varying from 140 cycles per second to 180 cycles per second, trying various samples of Silicone oil with viscosities ranging from 50 centistokes to 1000 centistokes at 25 degrees centigrade. This was accomplished by coating the absorber mass and housing with the oil while assembling and completely filling the absorber with oil. The absorber was then vibrated for several minutes to permit the oil to settle and oil was again added through a small filling connection at the top of the absorber housing.

The absorber was then oscillated at frequencies ranging from 25 to 210 cycles per second for a given position of fixed end pedestal. The natural frequency was determined from observations of the oscillograph. The point of maximum amplitude fixed the natural frequency.

While the system was vibrating at its natural frequency, the absorber mass was unlocked by turning the allen set screw on the top of the absorber. Amplitude readings were taken on the oscillograph to see if there was any reduction or increase in the amplitude. In practically all cases where the absorber mass was operating, there was a reduction in the amplitude at the natural frequency of the system (the natural frequency of the system with the absorber locked in position).

The Silicone oil was changed by dismounting the absorber assembly from the shaft and disassembling. After making a run, considerable difficulty was experienced in removing the absorber assembly from the shaft. The bore through the absorber assembly was designed to be a tight fit on the shaft to minimize the possibility of lost motion. The small sinusoidal oscillations imposed on the system had a fusing effect on the metal in contact with the shaft. Consequently, to remove the absorber from the shaft, it was necessary at each change of oil to press it off.

When the absorber was disassembled, it was thoroughly wiped with rags to remove every trace of oil. An oil of another viscosity was brushed on all the parts as they were being assembled. The same procedure was again followed in filling and testing the absorber.

Since the absorber mass was machined to permit a .005 inch clearance, the first series of runs were made with this clearance. After all the oils were tested, the springs were removed. The operation of absorber was then tested with the absorber mass locked and unlocked. The purpose of making the run with the absorber mass locked was to obtain an additional check so that the data on these series of investigations could be correlated with the data of the

first series. The frequency versus amplitudes curves with the absorber locked must be the same in both cases. Any discrepancy in these curves would indicate that the operating conditions for the subsequent runs are different from the initial runs.

After completing the investigation of all the Silicone oils it was found that DC-200, 50 centistoke oil was the most satisfactory viscous medium for a clearance .005 inches. The operating characteristics for the system at a natural frequency of 155 cycles per second are shown in Fig. 28. The absorber with springs at frequencies ranging from 100 to 210 cycles per second effectively reduces the amplitude of vibration. It accomplishes this by changing the natural frequency while in operation to 165 cycles per second. It is to be noted that the absorber when operating without springs accomplishes a small reduction in amplitude of the natural frequency of the system by shifting the natural frequency to 165 cycles per second but with a considerable increase in the amplitude at the new critical frequency. This marked increase in the amplitude in the case of the absorber operating without springs is believed to be due to the influence of transverse vibrations. There is no conclusive evidence to support a finding that it is due to either transverse or torsional vibrations. If it is due primarily to torsional effects, the ultimate result might be excessive shear stresses in the shaft. At any rate, the absorber would aggravate a condition that it was designed to correct.

The next step in the investigation was to machine the absorber mass to permit a clearance of .010 inches. The same procedure was

followed that was described above to test the operation of the absorber. It was found that the DC 200, 250 centistoke Silicone oil was the most suitable oil. In Fig. 29, and 31, the operation of the absorber is shown by a plot of the frequency versus the amplitude. This investigation disclosed that at a natural frequency of 160 cycles per second (Fig.31), the absorber with springs actually magnifies the amplitude of vibration. For a system of 170 cycles per second (Fig.29) the absorber with springs reduces the amplitude at the critical point. In each case the absorber operating with springs had a peak amplitude that was much greater than the peak amplitude the system operating with the absorber locked. For this condition of operation one effect of removing the springs from the absorber was simply to shift the natural frequency without any appreciable increase or decrease in amplitude of the peak point. This evidence seems to indicate that with increased clearances and more viscous oils, the absorber without springs is the more effective.

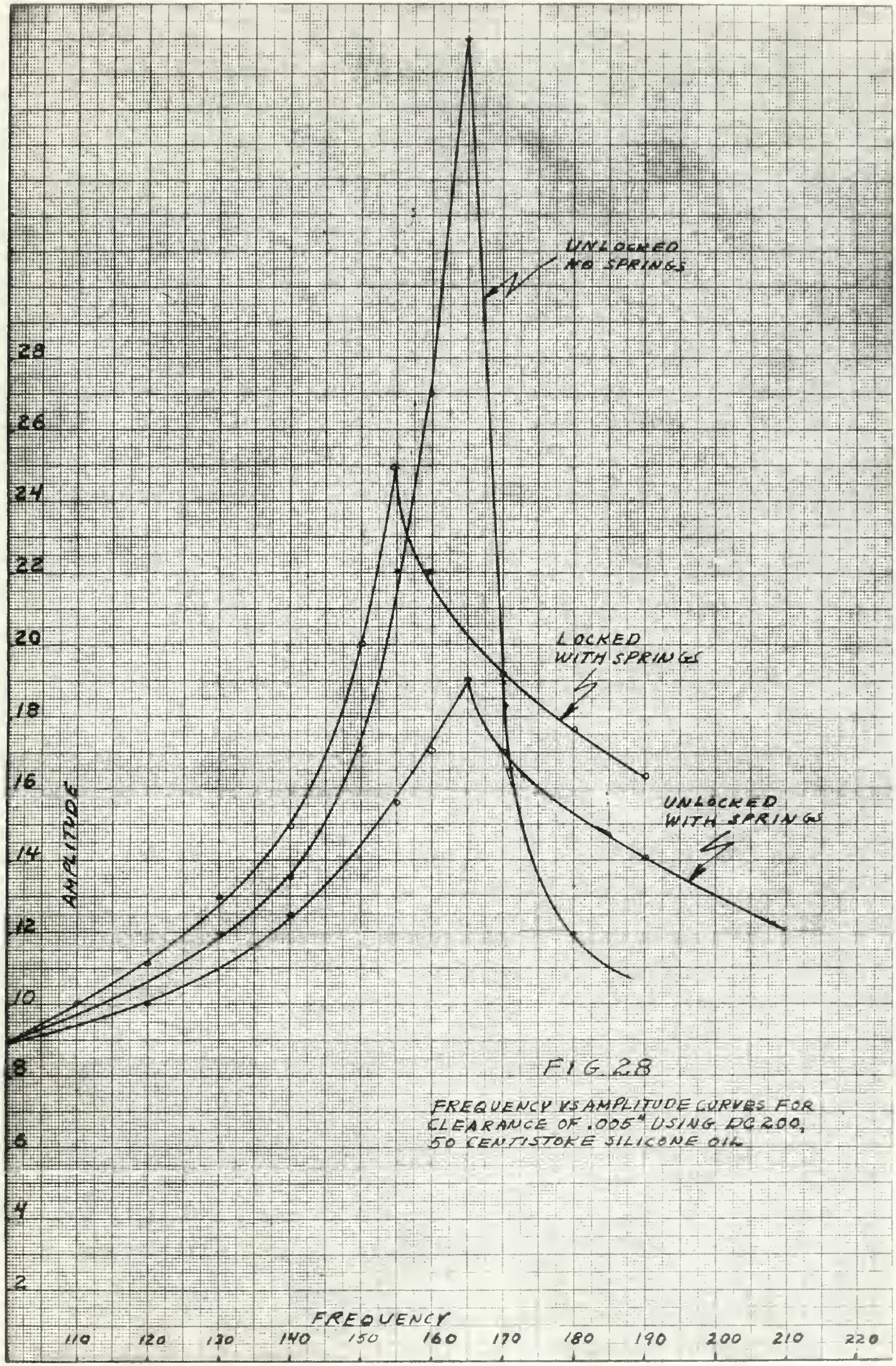


FIG. 28

FREQUENCY VS AMPLITUDE CURVES FOR CLEARANCE OF .005" USING DC 200, 50 CENTISTOKE SILICONE OIL

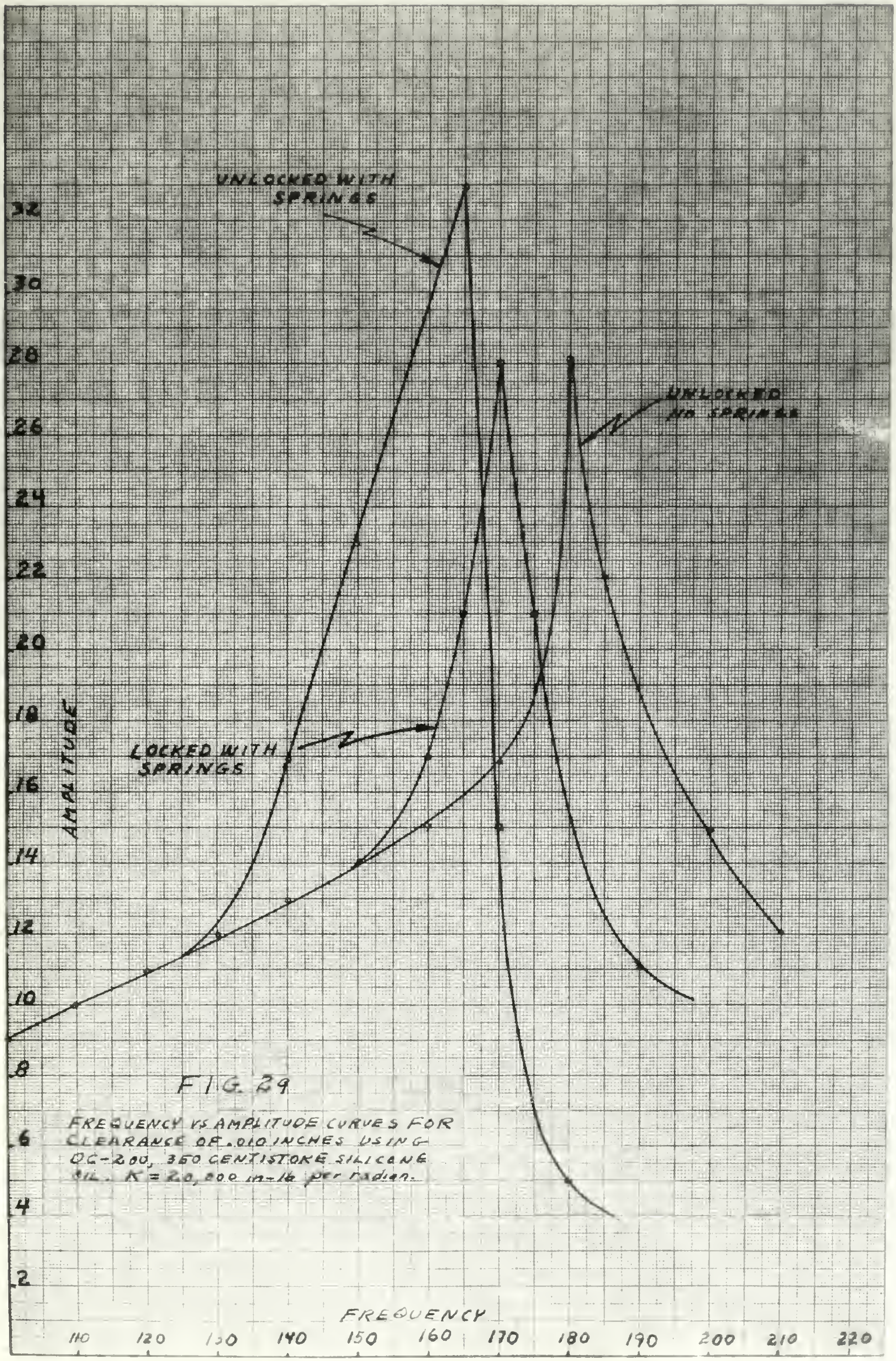


FIG. 29

FREQUENCY VS AMPLITUDE CURVES FOR CLEARANCE OF .010 INCHES USING DC-200, 350 CENTISTOKE SILICONE OIL. K = 20,000 in-lb per radian.

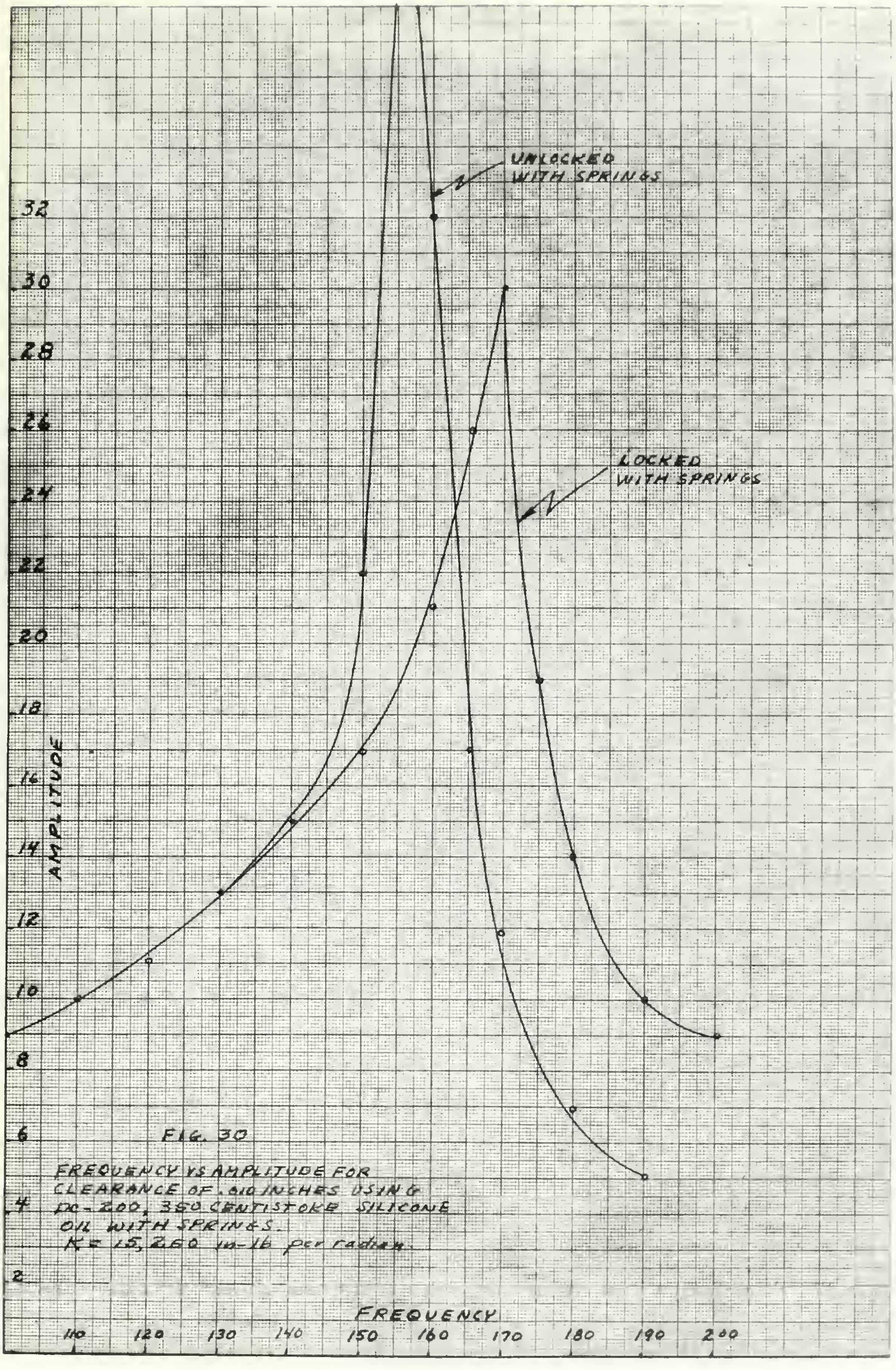


FIG. 30

FREQUENCY VS AMPLITUDE FOR
CLEARANCE OF .010 INCHES USING
DC-200, 350 CENTISTOKE SILICONE
OIL WITH SPRINGS.
K = 15,260 in-lb per radian.

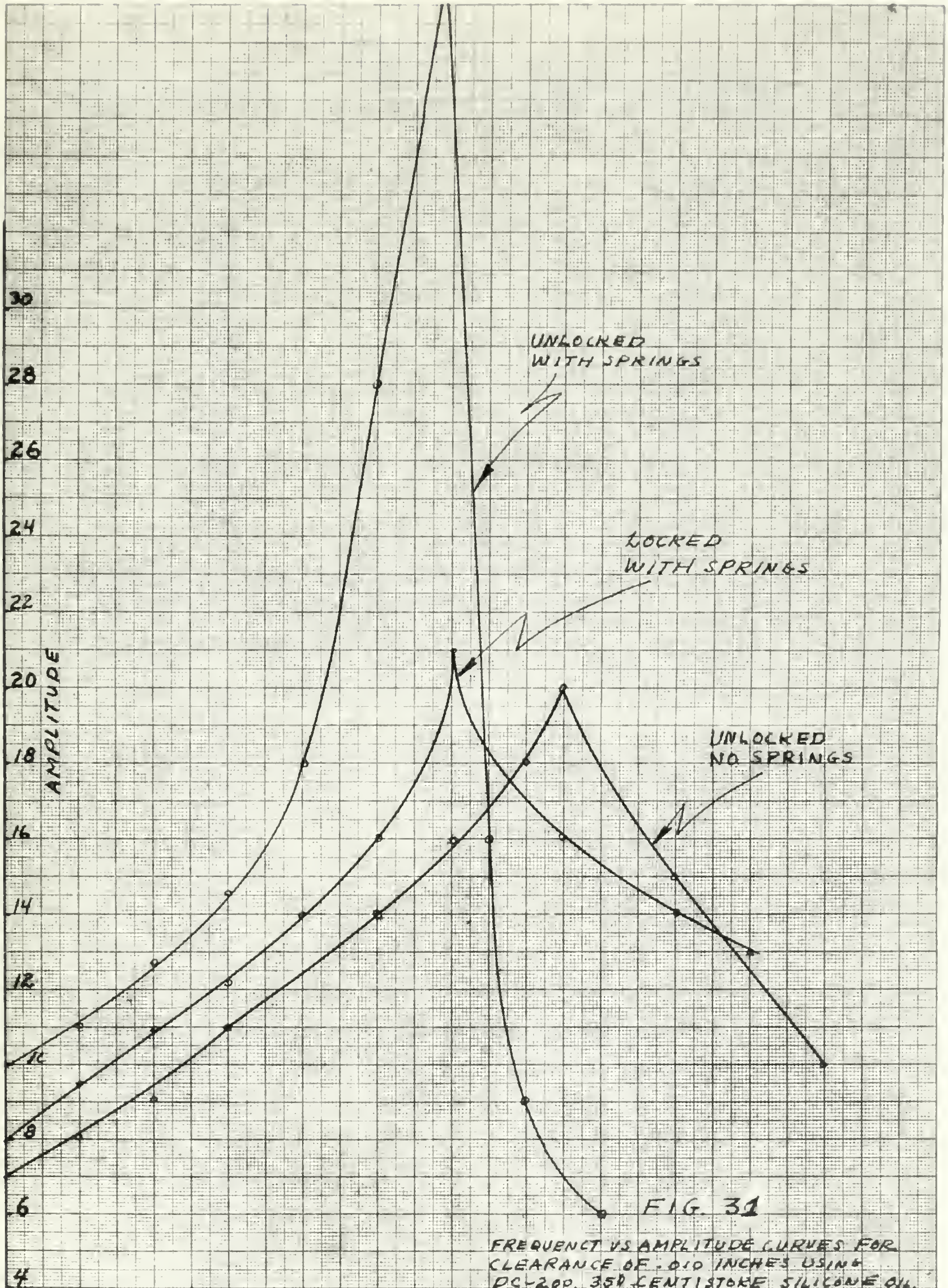


FIG. 31

FREQUENCY VS AMPLITUDE CURVES FOR CLEARANCE OF .010 INCHES USING DC-200, 350 CENTISTOKE SILICONE OIL. K = 31,300 in-lb per radian.

TABLE III

Experimental Data for Absorber Operating With Cantilever Springs, .005" Clearance Using DC 200-50 Centistoke Silicone Oil

Frequency (Cycles per Second)	Amplitude
100	9
110	9.7
120	11
130	13
140	14
145	15
150	20
155	25
160	22
165	20
170	19
180	17.5

TABLE IV

Experimental Date for Absorber Operating with Absorber Mass Locked in Position by Set Screw

Frequency (Cycles per Second)	Amplitude
100	9
110	9.4
120	10
130	11
140	12.5
150	14.2
155	15.5
160	17
165	19
170	17
180	15
190	14

TABLE V

Experimental Data for Absorber Operating Without Spring, .005" Clearance Using DC 200-50 Centistoke Silicone Oil

Frequency (Cycles per Second)	Amplitude
100	9
110	10
120	11
130	12
140	13.5
145	14
150	17
155	22
160	27
165	37
170	19
180	11
190	8

TABLE VI

Experimental Data for .010 inches Clearance using DC 200,350 Centistoke Silicone Oil without Springs, Absorber Mass unlocked. K = 31,300 in-lbs per radian.

Frequency (Cycles per Sec.)	Amplitude
100	7
110	8
120	9.5
130	11
140	12
150	14
160	16
170	17
175	20
180	18
185	16.5
190	15
195	13.5

TABLE VII

Experimental Data for .010 inches Clearance Using DC 200,350 Centistoke Silicone Oil with springs, Absorber Mass locked. K = 31,300 in-lbs per radian.

Frequency(Cycles per Sec.)	Amplitude
100	8
110	9.5
120	11
130	12
140	14
150	16
160	21
165	18.5
170	17
175	16
180	15.5
190	14

TABLE VIII

Experimental Data for .010 inches Clearance Using DC 200,250 Centistoke Silicone Oil with Springs, Absorber Mass unlocked K = 31,300 in-lbs per radian.

Frequency (Cycles per Sec.)	Amplitude
100	10
110	11
120	12.5
130	14.5
140	18
150	28
158	
165	16

170	9
175	7
180	6

Experimental Data for .010 inches Clearance using DC 200,350 Centistoke Silicone Oil with Springs, Absorber Mass unlocked. K = 15,250 in-lbs per radian

Frequency (Cycles per Sec.)	Amplitude
100	9
110	10
120	11
130	13
140	15.5
150	22
160	32
165	18
170	12
175	8
180	7
190	5

Experimental Data For .010 inches Clearance Using DC 200,350 Centistoke Silicone Oil with Springs, Absorber Mass locked. K = 15,250 in-lb per radian

Frequency (Cycles per Sec.)	Amplitude
100	9
110	10
120	11
130	13
140	15
150	17
160	21
165	26
170	30
175	19
180	14
190	10

TABLE IX

Experimental Data for Clearance of .010 inches using DC 200,350 Centistoke Silicone Oil with Absorber Mass unlocked, no springs. K = 20,000 in-lb per radian.

Frequency (Cycle per sec.)	Amplitude
100	9
110	10
120	11
130	12
140	13
150	14
160	15
170	17
175	19
180	28
185	22
190	15
200	

TABLE X

Experimental Data for Clearance of .010 inches using DC 200,350 Centistoke Silicone Oil with Absorber Mass locked. K=20,000 in-lb per radian. With Springs.

Frequency (Cycles per Sec.)	Amplitude
100	9
110	10
120	11
130	12
140	13
150	14
160	17
165	21
170	28
175	20
180	15
190	11

Experimental Data for Clearance of .010 inches Using DC 200,350 Centistoke Silicone Oil with Absorber Mass unlocked.
K = 20,000 in-lb per radian. With springs.

Frequency (Cycles per Sec.)	Amplitude
100	9
110	10
120	11
130	12
140	17
150	23
160	30
165	33
170	15
180	5

CHAPTER 4

CONCLUSIONS

The problem of solving the equations developed in Chapter II by a mechanical system poses many difficulties which have yet to be worked out. The principal difficulty lies in the fact that a purely torsional sinusoidal driving force applied to a rotating system is not readily attainable in a physical mechanical system. If this could be achieved, the allied problem of measuring the minute angular displacements of the rotating system would have to be considered. In this experimental investigation, the effects of rotation were completely ignored and the sinusoidal moment applied to a fixed torsional pendulum. Even in this idealized version of the basic problem, the effect of this transverse vibrations were an obstacle in the path of any experimental investigation. The author is firmly of the opinion that the experimental solution of the equations can be more readily obtained by the electrical analogue method. Such a solution would give us some criterion for the design of the absorber mass and its coupling shaft. However, the problem of experimentally proving any absorber design still presents a stimulating challenge to the design engineer. The way a vibration absorber functions is graphically brought out by the experimental data. It achieves a reduction in the amplitude of vibration by changing the natural frequency of a system and thereby changes the resonant condition. It also may create another resonant condition that may have a harmful effect on the mechanical system.

There has been no consideration given in the literature to the possible effect an absorber may have on the tranverse vibrations in a mechanical system. Although the findings in this investigation are not considered conclusive, there is some evidence that points to the fact that the absorber may magnify the effect of transverse vibrations. The author attributes the increased amplitude of the absorber operating without springs to the influence of transverse vibrations (See Fig.28). This aspect of the problem should not be ignored by the designer.

The variation of the natural frequency with changing amplitudes is a problem that merits further investigation. No attempt was made in this investigation to study this variation, but any factor that affects the natural frequency of a torsional multimass system is one of paramount concern to the absorber designer.

In the testing apparatus of this experimental investigation it was observed that the effect on the natural frequency of increasing or decreasing frictional resistance in bearings was appreciable. The possible variation of the natural frequency by bearing design is a consideration that should not be overlooked.

The author does not entirely concur in the current engineering practice of testing absorbers at certain critical frequencies of the engine. It is believed that the absorber should be checked at the critical frequency that it brings about during its operation to determine whether it is within the operating range of the engine. Some evidence has been adduced in this investigation to show that although absorber may be operating effectively at the natural frequency of the engine, it may introduce harmful amplitudes at

another frequency. The operation of the absorber should be tested at these frequencies.

It should be recognized in this discussion of the vibration absorber that is essentially a necessary evil that is appended to a complicated mass system and that it unquestionably further complicates that system. In essence, the design of an absorber for a particular system is a secondary method of avoiding torsional criticals. In concentrating on its design the primary and most important method of eliminating torsional critical should not be overlooked - that is, by designing the mass and elasticity of the system so that the critical frequencies will not be within the operating range of the system.

In the theoretical presentation of the problem the author has endeavoured by the derivation of a general solution to correlate and summarize the work of the various writers on this subject. It is hoped that this work will be of some assistance to the engineer who as yet as not been introduced to the many papers, both foreign and American, which discuss the many aspects of the design of the vibration absorber. The general theory outlined in this paper is undoubtedly more of academic interest than that of practical utility, for as other investigators have concluded, vibration absorbers are made and tested -- not designed.

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Thesis 15551
M34 Marciniak
A theoretical and experimental investigation of a tuned torsional vibration absorber

JE 24 '58 7509

MAY 6 306
MAY 6 7306
NO 20 57 620(F)
JE 24 '58 7509

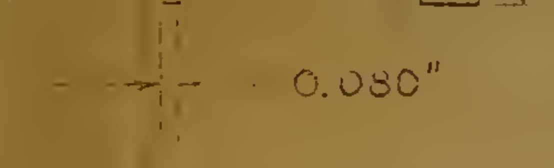
Thesis 15551
M34 Marciniak
A theoretical and experimental investigation of a tuned torsional vibration absorber.

Thesis
M34
Marciniak 15551
A theoretical and experimental investigation of a tuned torsional vibration absorber

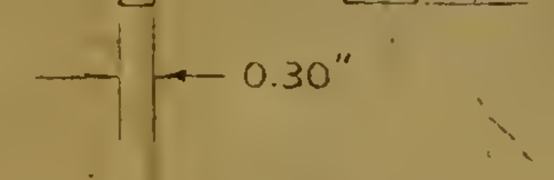
0.250"
0.5120"
2.0"
DRILL & TAP FOR #10-24-NC
HEX. HD. M. SC. 3/8" DEEP ON 30° SPACINGS
8.50"
7.5120"
8.0"
ABSORBER HOUSING
A
DRILL & REAM FOR 2#3 STANDARD
TAPERED PINS 7/8" DEPTH
ABSORBER ASSEMBLY
A
DRILL 10 HOLES FOR #10-24-NC
HEX. HD. M. SC. ON 30°SPACING
0.250"
DRILL 11 HOLES FOR #10-24-NC
HEX. HD. M. SC. ON 30° SPACING
0.250"
8.50"


ABSORBER HOUSING
ABSORBER ASSEMBLY
A
0.250"
DRILL 10 HOLES FOR #10-24-NC HEX. HD. M. SC. ON 30° SPACING
8.50"
.7505"
.7500"
5.0"
REAR COVERPLATE
0.250"
DRILL 11 HOLES FOR #10-24-NC HEX. HD. V. SC. ON 30° SPACING
8.50"
.7505"
.7500"
8.0"
FRONT COVERPLATE
DRILL 4 HOLES FOR 4-1/4" BOLTS EQUALLY SPACED ON 1" RADIUS
U.S. NAVAL POSTGRADUATE SCHOOL
ASSEMBLY & PARTS
TUNED VISCOUS
VIBRATION
ABSORBER
M.E. THESIS PROJECT
DRAWN BY: H.J.M.
SCALE: FULL SIZE
DATE: 9-15-50


CADMIUM PLATE & GRIND
0.5000"
0.040"R
0.30"
2.140"
0.080"
ABSORBER
SPRING
0.375"
1.75"
0.30"
AMPLITUDE
INDICATOR
7.5000"
3.510"
5.870"
0.60"
ABSORBER MASS
40.0"
0.8293"
0.8286"
0.7496
1.02"
SHAFT
0.5100"
0.0800"
0.0795"
DRILL 4 HOLES
EQUALLY SPACED
ON 1" RADIUS FOR
1/4-20-NC BOLTS



ABSORBER SPRING



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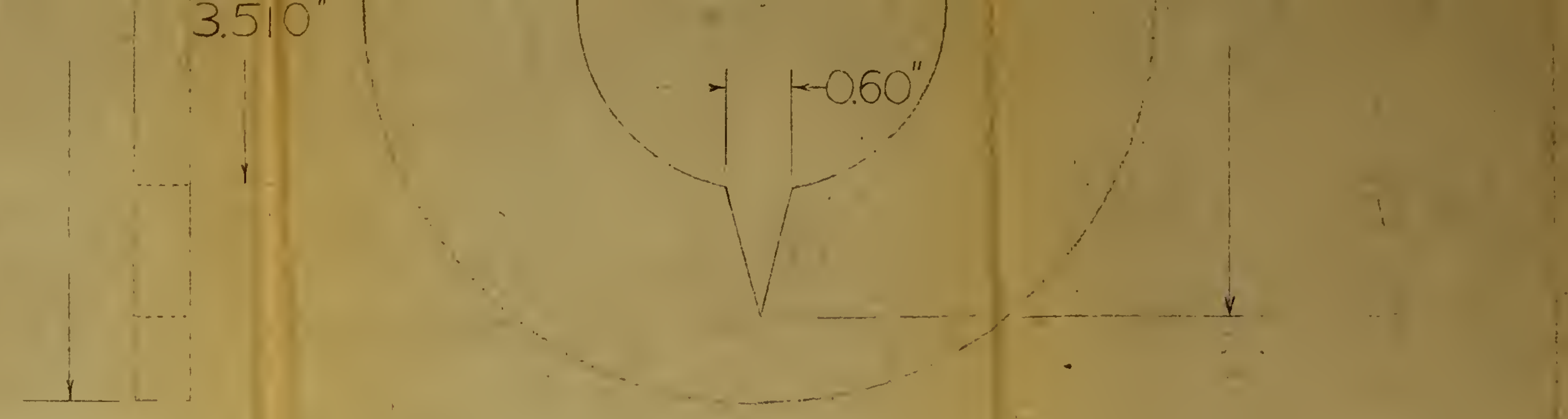
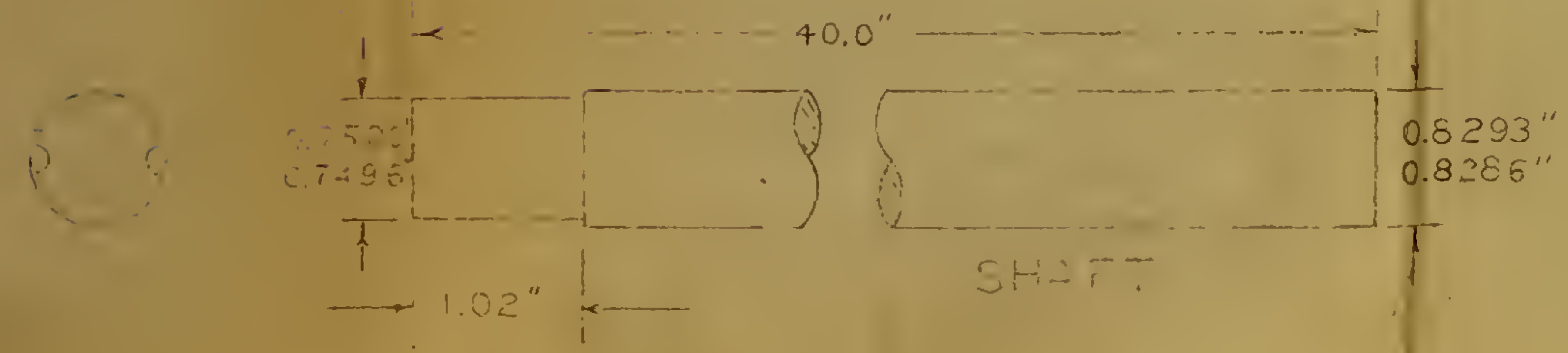
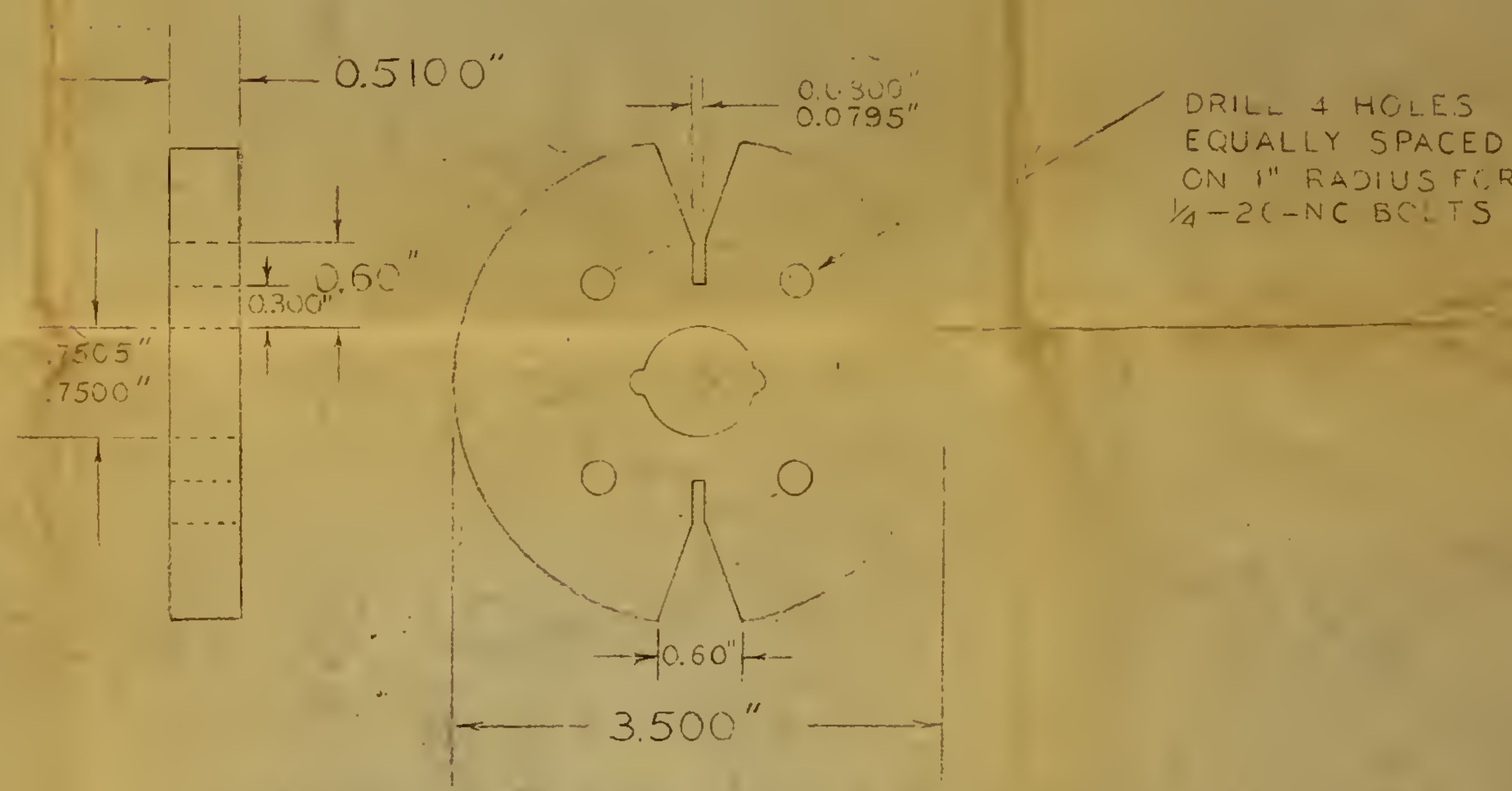


ABSORBER MASS

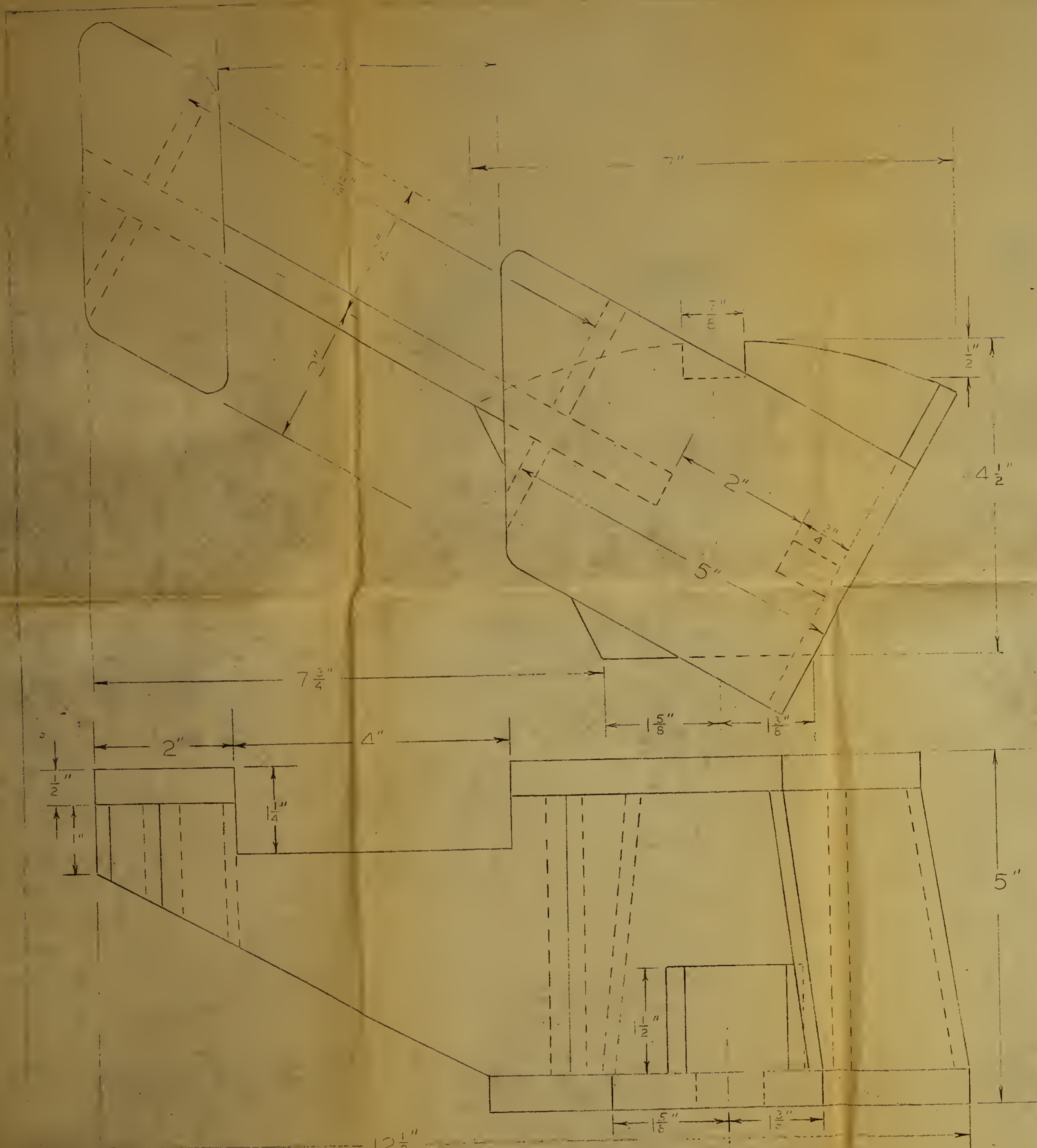




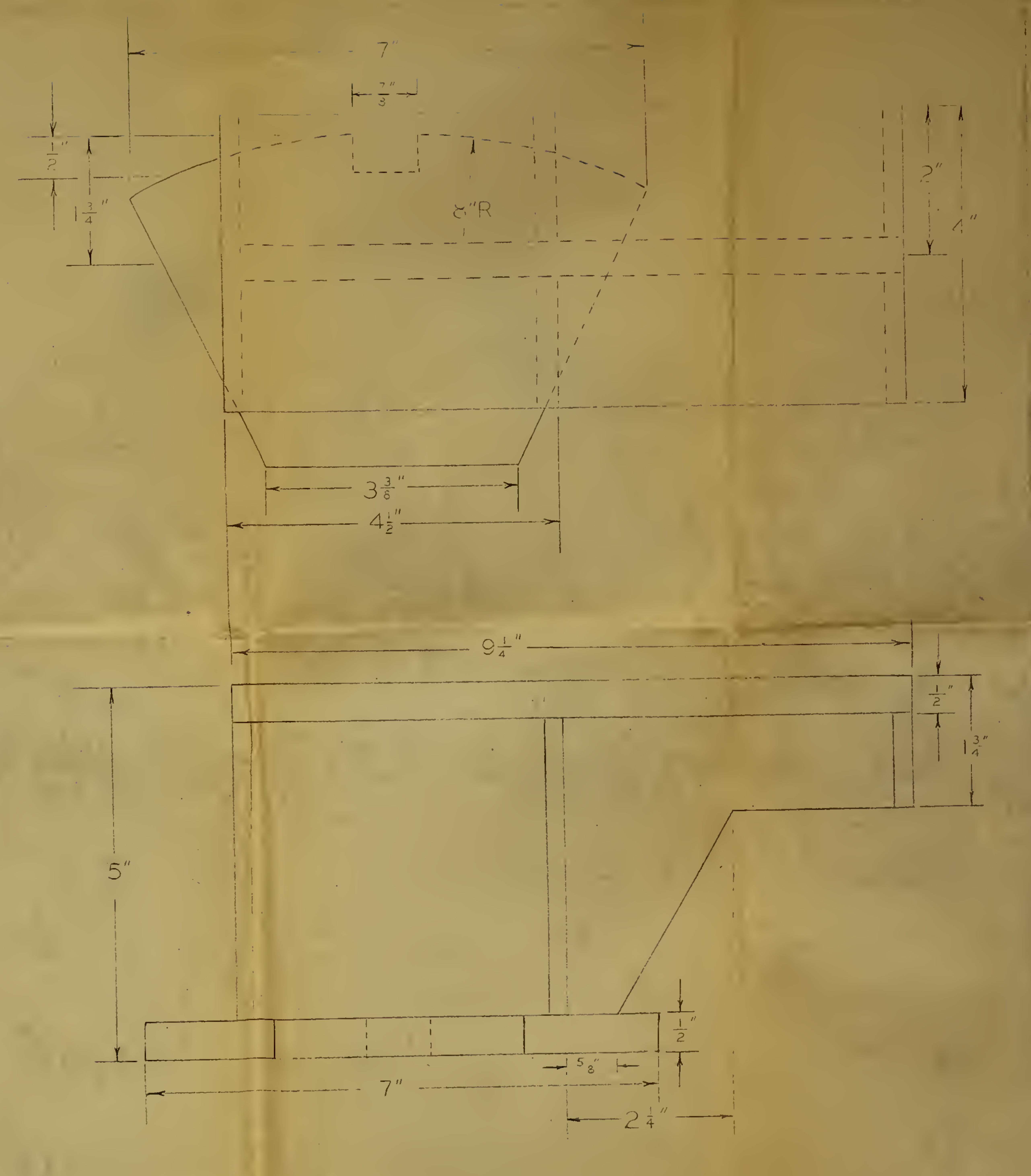
ABSORBER HOUSING CENTER PIECE

NO	NAME OF PART	MATERIAL	
1	ABSORBER HOUSING	SAE 1030	
1	" FRONT COVERPLATE	" "	
	" REAR "	" "	
1	" CENTER PIECE	" "	
1	" MASS-CADMIUM PLATE	" "	
1	" SHAFT	" 6150	
2	SPRING	" "	
2	#3 STANDARD TAPERED PINS		
1	AMPLITUDE INDICATER	STAINLESS	
1	DRIVING ARM	SAE 1030	
1	" " LOCKING PIECE	" "	
1	BUSHING	" 660	
1	" PEDESTAL	1/2" & 1/4" PLATE	
1	" CAP	SAE 1030	
1	FIXED END SUPPORT	1/2" & 1/4" PLATE	
1	" " CAP	SAE 1030	
1	DRIVER	" "	
1	" BUSHING	" 660	
1	" PIN	SAE 6150	
1	GUIDE PLATE	1/2" & 1/4" PLATE	
1	REAR SUPPORT	" " "	
1	F-CNT "	" " "	
1	EXTENSION STUD	SAE 1040	
2	HEX. SOCKET CAP SCREWS	1/4"-20-NC	1 5/16" LONG
4	" " BOLTS	" " "	1 1/4" "
2	" " CAP SCREWS	" " "	1" "
4	" " "	" " "	7/8" "
4	" " "	#10-24-NC	3/4" "
21	" " "	" " "	1/2" "
4	" " "	#12-24-NC	7/8" "

U.S. NAVAL POSTGRADUATE SCHOOL	
PARTS FOR	M.E. THESIS PROJECT
TUNED VISCOUS	DRAWN BY: H.G.M.
VIBRATION	SCALE: FULL SIZE
ABSORBER	DATE: 9-15-50



REAR SUPPORT

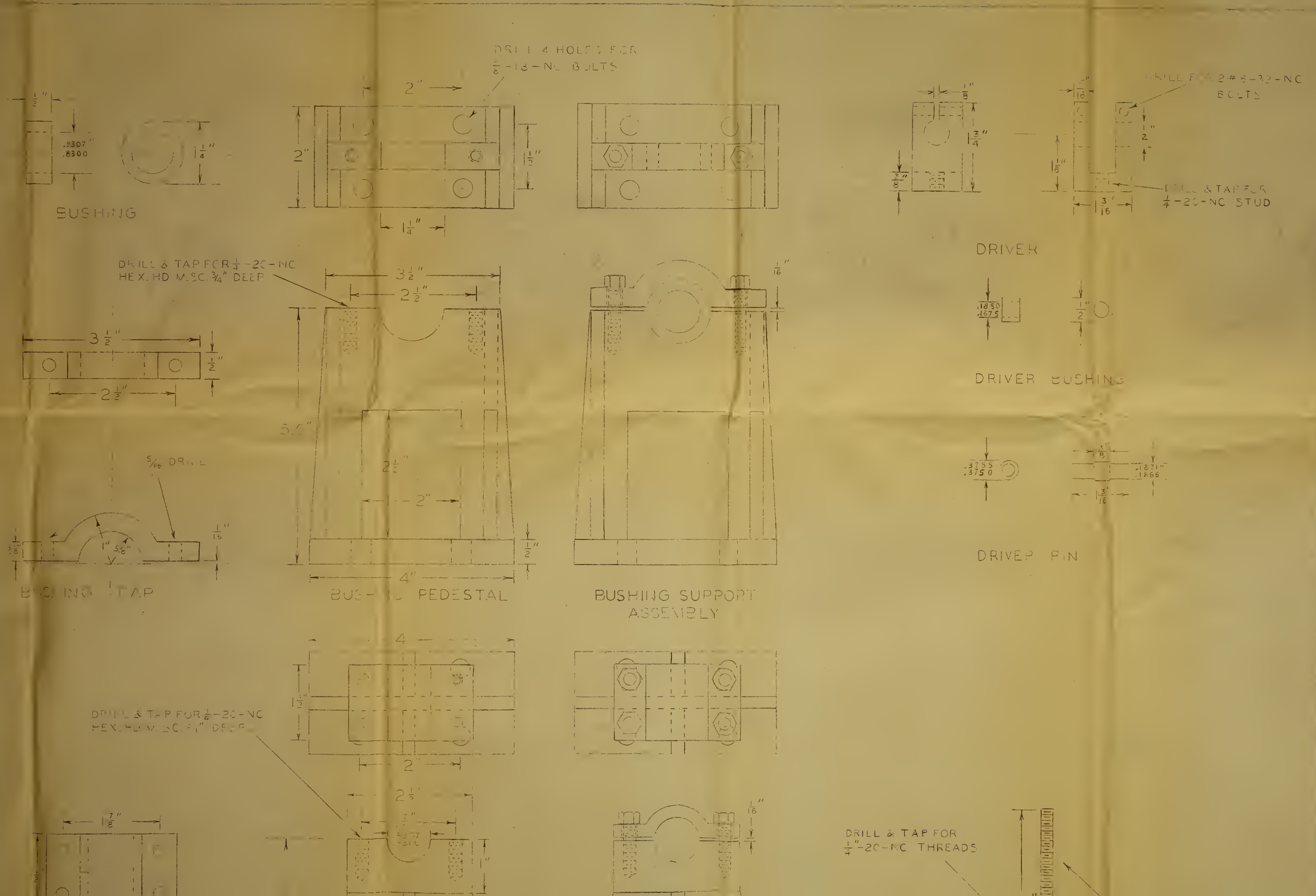


FRONT SUPPORT

45"

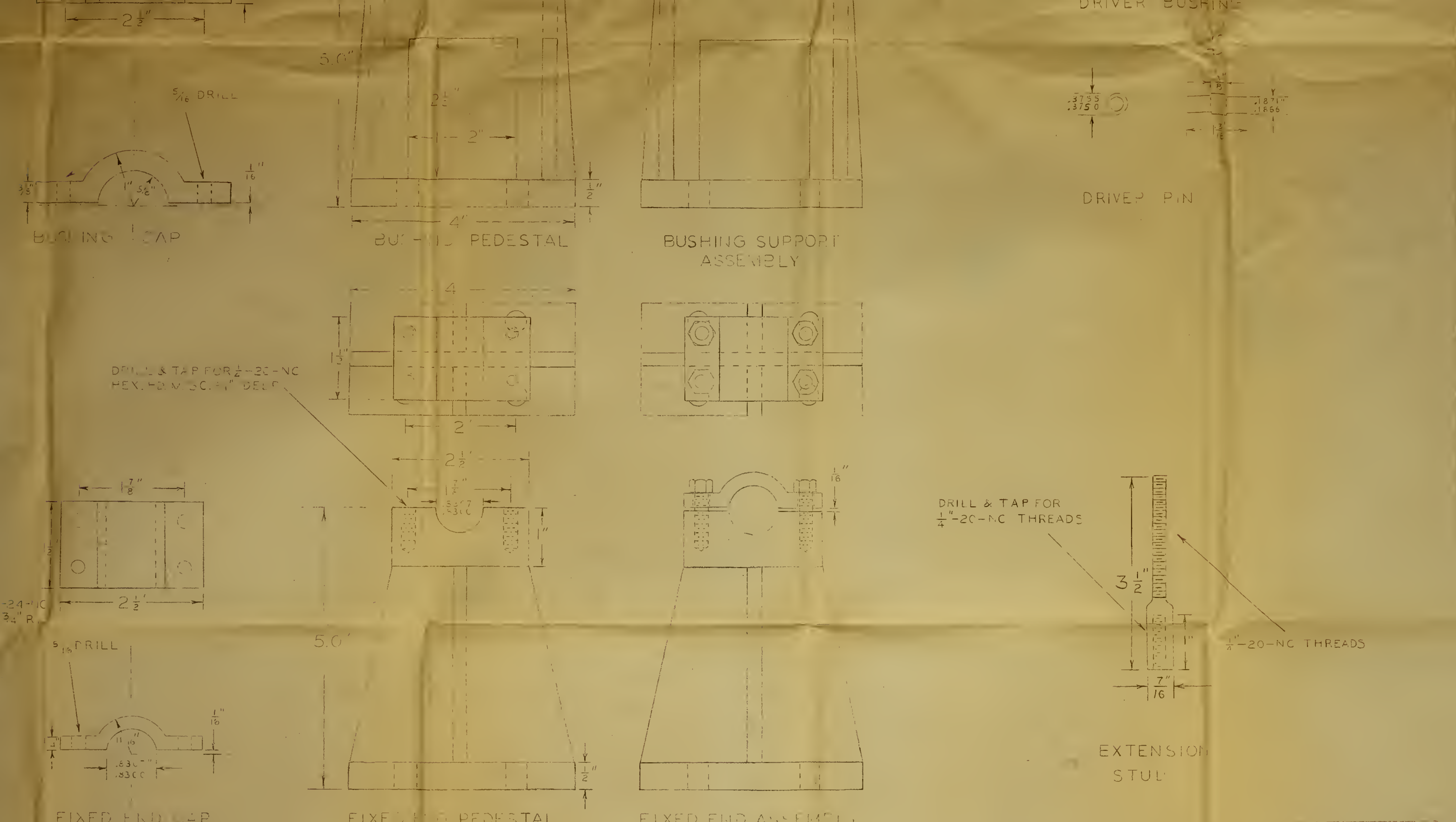
REAR SUPPORT
FRONT SUPPORT
GUIDE PLATE
SCALE: 1/2" = 1"
2"
4"
1/2"
1"
1 1/4"
5"
1 1/2"
1 5/8"
1 3/8"
12 1/2"
7"
5/8"
2 1/4"
45"
24 3/4"
21 3/8"
16 5/8"
13 3/8"
2"
13/32"
36"
4"
8 3/4"
3 7/8"
5 7/8"
10 7/8"
14 1/2"
18"
1"
1 1/4"
U.S. NAVAL POSTGRADUATE SCHOOL
SUPPORTS FOR TUNED VISCOUS VIBRATION ABSORBER
M.E. THESIS PROJECT
DRAWN BY: H.J. M.
SCALE: FULL SIZE EXCEPT AS INDICATED
DATE: 9-15-50

DRIVING ARM ASSEMBLY
BUSHING
DRILL & TAP FOR 1/4-20-NC HEX. HD M.SC. 3/4" DEEP
5/16 DRILL
BUSHING CAP
BUSHING PEDESTAL
BUSHING SUPPORT ASSEMBLY
DRIVING ARM



BUSHING
DRILL 4 HOLES FOR
BOLTS
DRILL & TAP FOR
HEX. HD M.SC. 3/4" DEEP
PEDESTAL
BUSHING SUPPORT
ASSEMBLY
5/16 DRILL
DRIVER
DRILL & TAP FOR
1/4-20-NC STUD
BOLTS
DRIVER BUSHING
DRIVER PIN
DRILL & TAP FOR
1/4"-20-NC THREADS


DRIVING ARM
5.0"
6½"
.840"
30°
DRILL FOR #12-24-NC HEX. SOCKET CAP SC. ON ¾" RADIUS
.3750"
.3745
5/16
BUSHING CAP
5/16 DRILL
1"
5/16"
3/8"
1/16"
BUSHING PEDESTAL
BUSHING SUPPORT ASSEMBLY
4"
2"
½"
DRILL & TAP FOR ¼-20-NC HEX. HD. M. SC. ¾" DEEP
2½"
1½"
1"
.8307
.8300
1⅞"
LOCKING PIECE
DRILL FOR ¼" BOLT
1"
1/16"
1 5/16"
30°
DRILL & TAP FOR #10-24-NC HEX. SOC. CAP SC. ON ¾" R.
DRILL & TAP FOR #12-24-NC HEX. SOC CAP SC. ON ¾" R.
½"
2¾"
.8307"
.8300
FIXED END CAP
5/16 DRILL
11/16"
FIXED END PEDESTAL
5.0"
FIXED END ASSEMBLY



U.S. NAVAL POSTGRADUATE SCHOOL	
SUPPORTS FOR	M.E. THESIS PROJECT
TUNED VISCOUS	DRAWN BY: H. J. M.
VIBRATION	SCALE: FULL SIZE
ABSORBER	DATE: 9-15-50